VALUE MANAGEMENT IN CONSTRUCTION

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The scope of knowledge presented in the manuals is necessary in activities of managers - construction engineers, managing undertakings in conditions of modern market economy. The manuals are approved by the European AEEBC association as a basis for recognizing manager qualifications. Modern knowledge in the field of management in construction, presented in the manuals, is one of prerequisites to obtain EurBE (European Building Expert) cards, a professional certificate documenting the qualification level of a construction manager in EU.

The manuals are designated for managers - construction engineers, students completing postgraduate studies “Management in construction” and students completing construction studies. Postgraduate studies are a recognized program, and graduates receive certificates recognized by 17 national organizations, members of AEEBC. The manuals were translated by Lingua Nova translation office.

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PART I

VALUE MANAGEMENT IN CONSTRUCTION
CHAPTER 1

INTRODUCTION

(A.MUKHERJEE, S.AKRAM, P.NOWAK)

Value Management (VM) and Value Engineering (VE) are techniques concerned with defining, maximizing and achieving “value for money” (VfM). These are systematic team based collaborative approaches, initially pioneered by the United States practitioners during the Second World War, to secure maximum output from limited resources.

At the initial stages of a project, VM provides an exceptionally powerful tool to explore a project’s objectives and aspirations from the client’s perspective.

Whilst the process originated in the manufacturing industry, the key initiating question (“what function does a component perform and how else can this function be performed”) is equally applicable to a wide range of disciplines, including construction and, within 10 years of the inception of this concept, the US Department of Defence incorporated VfM in the delivery of its very extensive construction programme.

Although its use was widespread in the US from the 1950’s, it took another three decades for this concept to be applied in the Europe, with its first ever application in the UK being in 1983 by the American company Xerox. One of the reasons for this slow uptake was probably due to the fact that the US approach to VfM required a separate team to audit the incumbent design team’s proposals. The design team was then expected to implement the audit team’s proposals. This predictably opened up numerous problems relating to design responsibility, legal complications and even professional rivalry. However, with the growing spread of the quality movement in the 1980’s and shift of the initial focus from materials to cost and quality, the concept was eventually accepted in UK and Europe, where the incumbent design team became a key part of the team undertaking the value audit.

With time, there was a realisation that materials, cost and quality as governed by the specification are not enough. Products had to respond to the needs of the customer. Emphasis therefore shifted to a more rounded appreciation of value,
encapsulating cost, time, performance, knowledge and technical competency (see fig. 1.1.). At the same time, the scope of the studies also have grown, to capture processes as well as products, with the focus being on expressing and measuring value in such a way that the project team can respond with the most effective solutions.

**Fig. 1.1. The evolution of value management**

A key differentiator between the VM concept and many other processes is that VM focuses on the expected outcome from a project. Only once the outcome is clearly established, understood, agreed and defined, does the process address the question of how it will be delivered. The aspired outcomes from a project is represented is a statement of the project objectives, expressed in terms of the expected benefits to the business. These are linked through the “value drivers” (defined as a functional attribute that is necessary to fully deliver the expected benefits from a project – equivalent to a primary function) to the design intent. Later, as the project evolves, these relate directly to the design solutions and what is built (see the value cascade, fig. 1.2).

---

1 Adapted from Kelly, J; Male, S and D Graham (2004) Value Management of Construction Projects, Blackwell Science
Fig. 1.2. The value cascade

CHAPTER 2

WHAT IS VM? DEFINITION
(A.MUKHERJEE, S.AKRAM, P.NOWAK)

Value relates to the assessment of the benefits brought by something in relation to the resources needed to achieve it. In the context of construction projects, it is normally expressed as a ratio between a function and the whole life cost for that function.

\[
\text{Value} = \frac{\text{Function}}{\text{Whole Life Cost}} \quad \text{or} \quad \frac{\text{What you get (or want)}}{\text{What you pay}}
\]

Thus, value can be increased by improved function or reduced whole life cost. Value for Money (VfM), as a concept, relates to the optimum balance between the benefits expected of a project and the resources expended in its delivery.

The three most common terms associated with the VfM concept are value management, value engineering and value analysis. Whilst they all are keys to the VfM concept, there are functional and systemic differences between these terms.

Value Management (VM) is about getting the right project, whilst Value Engineering (VE) is done to get the project right. Value Analysis (VA) relates to the improvement of a construction, manufacturing or management process and also to a post project review to establish value achievement.

The British/European Standards define VM as “a style of management, particularly dedicated to mobilise people, develop skills and promote synergies and innovation, with the aim of maximising the overall performance of an organisation” (BS EN 12973: 2000).

3 Further information and details about each process can be found at www.ivm.org.uk
An alternative functional definition can be “Value Management (VM), is a systematic and structured process of team based decision making. It aims to achieve best value for a project or process by defining those functions required to achieve the value objectives and delivering those functions at least cost (whole life cost or resource use), consistent with the required quality and performance” (Hamersley 2002).

The essential principles of VM place emphasise of itself being a continuous process within a structured framework.

An example of a VM study plan is shown in fig. 2.1.

---

**Fig.2.1. VM implementation plan**

---

2.1 UNDERSTANDING THE CONCEPT OF VALUE

Value is probably one of those things that everyone understands but no two people will describe it in the same way. It is not something tangible and is often quite difficult to measure or quantify. Yet, it is one of the key and perhaps most powerful concepts in the market. Different people view value differently. Value is subjective, prone to perceptions and pre-conceived notions. But for teams undertaking VM exercises it is absolutely essential that value is measured and quantified.

Early pioneers of VM identified three factors influencing value:

- **Utility** – Will it work effectively and do what it is expected to do? Most buildings are constructed in order to accommodate and support specific activities. The building will be judged a failure if it does not do this effectively. Thus, maximising the productivity of what is done within is a key component of the utility value in many buildings. A similar concept applies to civil structures such as roads and bridges. If a new power station is built, it must generate power reliably. In this case, the utility of the product is of primary importance, as is its ability to do so reliably. Thus, there may also be other secondary components of value.

- **Exchange** – can it be sold for a profit?

The property and real estate market is driven by the concept of exchange value. Exchange value relies on the fact that parties involved in the exchange have different values. The concept of value drivers enables project teams to optimise value for their projects. Normally this will involve trade-offs (exchanges) between different stakeholders to obtain the optimum balance between their differing values.

- **Esteem** – will it convey status or provide a “feel good” factor?

Esteem is a primary value for structures that need to convey an image or otherwise contribute to their environment. For example, corporate headquarters must convey to the public and clients alike what the corporation is about – that it is successful, it cares about details and it cares about its customers and things that its customers care about. In addition, the building must work as a building (utility value) and it must be saleable as an exit strategy (exchange value), but
the overriding importance is the esteem in which the outside world will hold the building and, by extension, its occupiers.
The relative importance of these three core types of value will vary depending on an individual’s perception of values.

2.2 MEASURING VALUE

In order to utilise the full effectiveness of the VM process, it is important that a quantification and measurement process is in place so that values can be quantified and measured. It is preferable to keep these measures objective and unambiguous. However, there may be instances where there may be an unavoidable element of subjectivity. In such cases, the VM team should make every effort to build a consensus, for example, by taking a series of observations from a number of stakeholders or by undertaking a survey.

In the construction industry there are a number of key performance indicators (KPI's) that are commonly used. These KPI's include elements such as client satisfaction – product and service, defects, cost predictability, time predictability, safety and so on. Most of the emphasis of these KPIs is on the process of delivering construction. By contrast, VM focuses on project outcomes, rather than the process for delivering project success. For this reason, the VM process utilises value drivers to quantify and measure value delivered by a project.

2.3 VALUE DRIVERS, VALUE PROFILE AND VALUE INDEX

A value driver is a functional attribute that is necessary to fully deliver the expected benefits from a project. In other words, it is a primary function i.e. a function that is directly related to the project objectives.
Table 2.1. below lists some of the key generic value drivers that are applicable to most construction projects.

Once the main functions (i.e. the value drivers) have been identified, it is beneficial to establish the hierarchy of importance between the main value drivers, thus establishing the *value profile* of the project.

**Table 2.1. Generic value drivers**

<table>
<thead>
<tr>
<th>Value driver</th>
<th>Key prompt question</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enhance/achieve desired financial performance (of the structure)</td>
<td>Is the structure affordable?</td>
</tr>
<tr>
<td>Manage the delivery process effectively (maximise project delivery efficiency,</td>
<td>Are the project management processes efficient?</td>
</tr>
<tr>
<td>minimise waste)</td>
<td>Are the right people engaged at the right time?</td>
</tr>
<tr>
<td></td>
<td>Is the delivery chain effectively managed?</td>
</tr>
<tr>
<td></td>
<td>Are the resources used effectively?</td>
</tr>
<tr>
<td>Maximise operational efficiency, minimise operational costs</td>
<td>Does the structure work well for the end users?</td>
</tr>
<tr>
<td>Attract and retain employees/occupants/users</td>
<td>Is it a nice place to live/work/be?</td>
</tr>
<tr>
<td>Protect the appropriate image</td>
<td>Does the structure convey the appropriate image?</td>
</tr>
<tr>
<td>Minimise maintenance costs</td>
<td>Is the structure easy to maintain?</td>
</tr>
<tr>
<td>Enhance the environment</td>
<td>Is the structure environment friendly?</td>
</tr>
<tr>
<td></td>
<td>Is the structure built using the ethos of environmental sustainability?</td>
</tr>
<tr>
<td>Comply with third-party constraints</td>
<td>Does the structure conform to legal and other external stakeholder requirements?</td>
</tr>
<tr>
<td>Ensure health and safety during implementation, operation and occupation</td>
<td>Is the structure safe to construct and operate?</td>
</tr>
</tbody>
</table>

---

Assignment of quantitative measures (also known as setting metrics) to each individual value driver and agreeing performance measure (see fig.2.2.) will enable the VM team to assess and quantify performance and thus generate the value index of the project. An example of this is shown in Table 2.1.

![Value measurement process](image)

**Fig.2.2. Value measurement process**

Successive reassessments of the value index after each value study can give the project team a clear indication of how effective their efforts have been and where additional effort is needed to further improve value.

It is important to consider potential difficulties in agreeing objective metrics for attributes that may be somewhat subjective. Ideally, a metric should be identified that cannot be influenced by the observer. For example, costs and quantities of materials can be estimated from the market and are therefore objective. Even relatively soft values can be the subject of objective metrics. For example, the level of satisfaction of the end users of a department can be

---

measured by the number of letters of complaints received. Where such objective metrics are not possible, it may be necessary to resort to surveys, ideally undertaken by independent observers.

Table 2.2. Example of value profile

<table>
<thead>
<tr>
<th>Value driver</th>
<th>Importance weight(%)</th>
<th>Metric</th>
<th>Performance (1 – poor, 10 - excellent)</th>
<th>Weighted value score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial</td>
<td>15</td>
<td>Cost</td>
<td>3</td>
<td>45</td>
</tr>
<tr>
<td>Project management</td>
<td>15</td>
<td>KPI</td>
<td>4</td>
<td>60</td>
</tr>
<tr>
<td>Business efficiency</td>
<td>20</td>
<td>Output</td>
<td>4</td>
<td>80</td>
</tr>
<tr>
<td>Image</td>
<td>10</td>
<td>Survey</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>Occupation cost</td>
<td>15</td>
<td>Cost</td>
<td>3</td>
<td>45</td>
</tr>
<tr>
<td>3rd party requirements</td>
<td>10</td>
<td>Audit</td>
<td>8</td>
<td>80</td>
</tr>
<tr>
<td>Sustainability</td>
<td>15</td>
<td>Output</td>
<td>6</td>
<td>90</td>
</tr>
<tr>
<td>Total value index(^7)</td>
<td>(100%)</td>
<td></td>
<td></td>
<td>420</td>
</tr>
</tbody>
</table>


\(^8\) As a general principal, a score of 850 can be taken as excellent; 750 to be good; 500 implies room for improvement and less than 500 requires improvement
CHAPTER 3

TECHNIQUES FOR VALUE MANAGEMENT
(A.MUKHERJEE, S.AKRAM, P.NOWAK)

There are a number of techniques which are commonly used in the conduct of VM studies. Some of these techniques are:

1. Function analysis
2. Function analysis system technique (FAST)
3. Cost/worth
4. SMART methodology
5. Value drivers
6. Value benchmarking (or value profiling)
7. Options selection
8. Weighting techniques
9. Creative techniques
10. Evaluation techniques
11. Scenarios technique
12. Target costing
13. Function performance specification (FPS)
3.1 FUNCTION ANALYSIS

Function analysis provides one of the defining characteristics of VM and differentiates it from many other problem solving techniques. It is a method for analysing the functions of the constituent parts of a project.

There are many approaches to function analysis, some very structured (such as the function system analysis technique – see 3.2) and others are less formal (such as value trees or mind maps).

One of the key principles of the VM process, as stated before, is that it focuses on achieving successful outcomes rather than on the process of getting there. Functional analysis provides a very powerful tool to identify intended outcomes. In developing a functional model, the process team is forced to make a very clear definition of the project by considering key questions such as:

- What are we trying to achieve?
- What must we get right if we are trying to achieve it?
- What considerations do we need to bear in mind while designing it?
- How do various design solutions contribute towards achieving the desired outcome?

The method relies on developing a function cost matrix in which the costs of performing each of the identified functions may be determined by allocating the elemental costs across the functions. An example of this is shown in Table 3.1. One of the techniques used to describe each function is by using an active verb, a measurable noun and a qualifying phrase or adjective. Table 3.2. lists out some of the commonly used active verbs, adjectives and nouns for describing functions.
Table 3.1. Function cost matrix\(^9\)

<table>
<thead>
<tr>
<th>Element</th>
<th>Cost</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>250</td>
<td>100</td>
<td>25</td>
<td>25</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>4</td>
<td>300</td>
<td>150</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>5</td>
<td>75</td>
<td>75</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>500</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>7</td>
<td>250</td>
<td></td>
<td>200</td>
<td>50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>150</td>
<td>75</td>
<td></td>
<td></td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td>1675</td>
<td>275</td>
<td>400</td>
<td>375</td>
<td>325</td>
<td>300</td>
</tr>
</tbody>
</table>

Table 3.2. Active verbs, qualifiers and measurable nouns to describe functions\(^{10}\)

<table>
<thead>
<tr>
<th>Active verb</th>
<th>Qualifiers</th>
<th>Measurable noun</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximise</td>
<td>Abnormal</td>
<td>Requirements</td>
</tr>
<tr>
<td>Minimise</td>
<td>Third party</td>
<td>Energy</td>
</tr>
<tr>
<td>Satisfy</td>
<td>Legal</td>
<td>Ambience</td>
</tr>
<tr>
<td>Create</td>
<td>Corporate</td>
<td>Productivity</td>
</tr>
<tr>
<td>Display</td>
<td>Working</td>
<td>Comfort</td>
</tr>
<tr>
<td>Enable</td>
<td>Conforming</td>
<td>Environment</td>
</tr>
<tr>
<td>Prevent</td>
<td>Consistent</td>
<td>Image</td>
</tr>
<tr>
<td>Enhance</td>
<td>Best in class</td>
<td>Performance</td>
</tr>
<tr>
<td>Diminish</td>
<td>World class</td>
<td>Facility</td>
</tr>
</tbody>
</table>


\(^{10}\) Adapted from Kelly, J. Male, S. and D. Graham (2004)
3.2 FUNCTION ANALYSIS SYSTEMS TECHNIQUE (FAST)

This technique relies on logically linking functions and allows people from different technical backgrounds to use a common language to describe and link the functions of complex systems, using non-technical knowledge, to build a FAST diagram. In order to produce the FAST diagram, the team will have to interact and communicate with one another effectively to arrive at a logical diagram that they all can understand and agree with. A simple FAST diagram for part of an office building is illustrated in Diagram 3.1.

How?

| Project Image | Enhance reception hall | Form new entrance | Extend existing entrance |

Why?

Fig. 3.1. FAST diagram for part of an office building

The how/why logic provides the key to developing a logic-linked function diagram. Randomly generated functions (as used in function analysis) can be logically linked through the use of questions “how” and “why”. The level of abstraction, from left to right, gradually diminishes thus providing lower levels of abstraction towards the right hand side of the diagram. This is also reflecting “dependencies”, where a function of lesser abstraction is dependent upon higher-level abstraction functions. Thus, changing the higher order functions will effectively change the outcome. While looking for innovations or improvement ideas, it is therefore necessary to address functions of higher-level abstractions.

In addition to the “how” and “why”, other key words that are used for producing FAST diagrams include “when” (to identify things that occur simultaneously with an identified function – see diagram 3.2.), “and” (to indicate when two dependent functions happen simultaneously) and “or” (to describe that eventually either one dependent function will happen or another, depending upon circumstances) – see diagram 3.3.

### 3.3 COST/WORTH

A key principle of VM is to analyse each of the functions and to assess what it actually costs to perform the function, using the functional cost matrix (Table 3.1. in section 3.1). For comparison, the team assesses the lowest cost at which the function can be performed, referred to as the function’s “worth”. A comparison of the two Diagrams gives an indication of the “cost/worth” of the function being examined. The functions whose cost significantly exceeds their worth may warrant further study to explore whether they can be performed in a different way at less cost. However, this approach must be understood with the caveat that it is absolutely critical that all functions performed by each component must be taken into account. For example, the basic function of granite paving may be to support pedestrians. This function can also be performed by using concrete paving slabs at a much lower cost. However, there are other functions that the granite paving will have contributed to, not least of which is the aesthetics of the area. Similarly, granite performs an additional function “resist wear” considerably better than its concrete equivalent. It is vital, therefore, that in considering basic function of a component, the other functions to which it contributes are taken into account to assess its true “worth”.

22
Fig. 3.2. FAST diagram utilising “When” logic\textsuperscript{12}

\textsuperscript{12} Adapted from Kelly, J. Male, S. and D. Graham (2004)
3.4 SMART METHODOLOGY

The SMART (Simple Multi-Attribute Rating Technique) methodology, introduced in the mid-1990’s, generated two evolutionary concepts from FAST methodology. The first of these was the value tree. Instead of expressing the essential components necessary to fulfil the project objectives by means of two (or even with the third qualifier) word functions, this methodology uses the concept of a value tree to link functions. The second innovation was to create an importance hierarchy for the functions.

The value tree is similar to the FAST diagram in that it normally begins on the left hand side with a statement of project objectives. The answer to the question “how” is expressed in simple value-adding attributes required to deliver the project objective. Each of these is broken down into the attributes that add value to that branch, thus building a tree of decreasing abstraction from left to right. Fig.3.4. illustrates an example of a value tree for a school.

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Fig. 3.4. Example illustration of a value tree for a school

Fig. 3.5. A SMART value tree for a school with importance weights\textsuperscript{15} \textsuperscript{16}

\textsuperscript{15} Note: the functions that are not expanded have not been taken into account in the weighting

\textsuperscript{16} Adapted from Kelly, J. Male, S. and D. Graham (2004)
The second aspect of the SMART method requires the client to rank the value adding attributes in order of their importance in achieving project objectives. Under the SMART value tree the weighting for each of the values is expressed as a decimal of less than 1, such that for any level of abstraction for any single attribute, the total weighting adds to 1. This is illustrated using the same value tree as in fig. 3.5.

This methodology enables the VM team to undertake two tasks. By attributing costs to the value tree (by distributing the cost of individual elements across the branches of the tree to which they contribute), it is possible to assess the relative costs to undertake a function with its importance in the overall project, and open up the perspective of contemplating different approaches to undertake the same function for fewer resources.

### 3.5 VALUE DRIVERS

Many of the functions in construction projects are not physical in nature. Since these abstract functions are essential to adequately describe a project, the concept of value driver, instead of primary function, is often used. Value drivers are those things which contribute to the value of the building and are readily understood.

Some commonly used value drivers are illustrated in the Table 3.3. below.
Table 3.3. Generic value drivers for construction projects\textsuperscript{17}

<table>
<thead>
<tr>
<th>Value driver</th>
<th>Addresses the question</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enhance/achieve desired financial performance</td>
<td>Is the project affordable?</td>
</tr>
<tr>
<td>Manage the procurement process effectively (maximise project delivery efficiency, minimise waste)</td>
<td>Are the project management processes efficient? Are the right people engaged? Is the supply chain effectively managed? Are the resources used efficiently?</td>
</tr>
<tr>
<td>Maximise operational efficiency, minimise operational costs</td>
<td>Does the end product of the project work well for the end users?</td>
</tr>
<tr>
<td>Attract and retain employees/users</td>
<td>Is it fit for its intended use and purpose?</td>
</tr>
<tr>
<td>Project the appropriate image</td>
<td>Does the end product convey the appropriate image?</td>
</tr>
<tr>
<td>Minimise maintenance costs</td>
<td>Is the end product easy to maintain?</td>
</tr>
<tr>
<td>Enhance the environment</td>
<td>Is the project and the end product environment friendly? Is it constructed using the principles of environmental sustainability?</td>
</tr>
<tr>
<td>Comply with third party constraints</td>
<td>Does the project and the end product have to conform to particular legal and other external stake holder requirements?</td>
</tr>
<tr>
<td>Ensure health and safety during project implementation and in operation</td>
<td>Is the end product safe to construct and use?</td>
</tr>
</tbody>
</table>

Development of generic value drivers offers an advance on the generation of random value drivers for each and every project. While random value drivers may just be as effective for the purposes of VM, the resulting function analysis would be unique to the particular project and the circumstances in which they were developed. Use of generic value drivers creates the advantage of comparability, where projects with similar objectives can be benchmarked and compared against one another.

\textsuperscript{17} Adapted from Dallas (2006)
3.6 VALUE BENCHMARKING (OR VALUE PROFILING)

Utilising the value driver tool, it is possible to describe the client’s value priorities, which forms the project’s value benchmark or value profile. This, in turn, creates a way of identifying those parts of the project which provide most potential for adding value and shaping the project in the right direction.

Value benchmarking should take place at the beginning of a VM study. Utilising the weighted value drivers, the client team (excluding client consultants and advisers, but including the end users) produces the value profile. The VM team, in conjunction with the client team, creates an acceptable range for each value driver using objective matrices (for example, using a scale of 1 to 10, where 1 is unacceptable and 10 equals delight).

Table 3.4 illustrates an example of value benchmarking (or profiling).

The VM team, with the client team, may also identify targets within each range for the VM objective to achieve. These targets will help to benchmark the exercise and the project, by clearly contrasting current performance values against the target (benchmark) values.
Table 3.4. Value benchmarking (or value profiling)\textsuperscript{18}

<table>
<thead>
<tr>
<th>Importance weighting</th>
<th>Attract &amp; retain staff</th>
<th>Facilitate access</th>
<th>Encourage shopping</th>
<th>Comply with tenant requirements</th>
<th>Minimise operating cost</th>
<th>Complete on time</th>
<th>Complete on budget</th>
<th>Value index</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
<td>15</td>
<td>25</td>
<td>5</td>
<td>5</td>
<td>25</td>
<td>15</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>Performance (1-10)</th>
<th></th>
<th>Value score (Target)</th>
<th></th>
<th>Value score (Target)</th>
<th></th>
<th>Value score (Target)</th>
<th>Value score (Target)</th>
<th>Value score (Target)</th>
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<td>7</td>
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<td>8</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>Current profile</td>
<td>Performance (1-10)</td>
<td></td>
<td>Value score (Actual)</td>
<td></td>
<td>Value score (Actual)</td>
<td></td>
<td>Value score (Actual)</td>
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<td>-15</td>
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<td>-100</td>
<td>-45</td>
<td>-360</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\textsuperscript{18} Adapted from Dallas (2006) – items in bold have the highest priority and greatest value deficit and hence require immediate focus
3.7 OPTIONS SELECTION

A weighted value driver model provides an objective way of making decisions. By using an options evaluation matrix, it is possible to assess the relative benefits of each option using the weighted value drivers as evaluation criteria. Needless to say that the best option will be the one that satisfies the value drivers the most. A value score for each option is obtained by multiplying the weighting of each value driver by the degree to which the options satisfy the. It is recommended that a scale of 1 to 4 is used, where 1 is poor and 4 is excellent. It is better to use a 4 point score than a more common 5 point scale, as there may be a tendency for groups to pick the midpoint number 3 on the 5 point scale, whereas on a 4 point scale the group is forced to select above or below the mean by choosing 3 or 2. Adding all the value scores across all functions for each option gives a total value score for that option. The option with the highest score is that which best satisfies the requirements in the project objectives.

3.8 VALUE FOR MONEY

Value for Money (VfM) is a further sophistication of this technique, used for option selection. VfM is calculated by dividing the value score by total cost of the option. This helps to differentiate between two options, each of which has a high value score, but where one costs significantly more than the other.

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19 It is recommended that a sensitivity analysis be performed as well, since small changes in the weighting of each option against the value drivers will have an impact on the end result. Even when scientific methods are used to assess weightings or assessing how well each option satisfies a particular function, there remains an aspect of subjectivity. A sensitivity analysis should be performed, varying both the importance weightings and satisfaction assessments, in order to ensure that the process is robust.
3.9 WEIGHTING TECHNIQUES

There are a number of different weighting techniques with varying degrees of complexity which are commonly used in VM studies. These include:

- Dots;
- Distribution of points; and,
- Paired comparisons.

*Dots* is perhaps the most simplistic technique, least scientific but quickest method of indicating the relative importance between a number of items, where the VM team has to allocate dots to their preferred choices. There should be some guidance as to how many favourites to select. The number should not be more than one third of the items under consideration. The item with the most dots is ranked most important and so on.

A more statistically sound way to assign weightings is to ask the team to allocate a fixed number of points between the items (*distribution of points*). Each person can put as many points as they wish against any one item, but must use all their points and no more. At the end the points against each item are added up, and dividing this by the total number of contributors gives an average score for each item. Then these scores are normalised to arrive at a percentage weighting for each.

However, the outcome of this technique can potentially be biased if the VM team does not have representatives from the whole project team. Furthermore, sometimes there may be a significant divergence in individual weightings across the team. In this situation, the average will not be representative of all the members’ views. Convergence tools such as Delphi technique\(^\text{20}\) or similar can be utilised in these situations.

A further sophisticated technique is paired comparison, where direct comparisons between each of the attributes being weighed are made. In this technique, each item is judged against one another. Scales of assessing by how much one attribute is better than another vary. A 3-point scale (1 being low and

3 being high) is most commonly used. The score against each attribute is calculated by adding up the total number of items that attribute appears in the matrix, with each entry being multiplied by the scale factor if applicable. The weighting is then calculated by normalising the scores on a percentage basis.

### 3.10 CREATIVE TECHNIQUES

One of the fundamental concepts underpinning VM is to encourage and extract innovative solutions. Functional analysis can be utilised to generate and foster innovation and creativity.

Ideas can be generated simply by asking questions such as “how can the component be made better, efficient and cheaper”, particularly in terms of value drivers. This is based on the fundamental assumption that there will always be more than one way to solve any problem.

Creative techniques such as brainstorming can be used to generate and foster ideas.

It is also necessary to create evaluation processes to evaluate the ideas generated, by creating common evaluation criteria, so that the ideas can be ranked and suitable implementation proposals can be developed.

It may also be necessary to undertake scenario appraisals to risk assess the selected ideas.

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3.11 TARGET COSTING

VM techniques can also be utilised to establish not only what functionality the client wants, but also how much the client is willing to pay for it.

Construction, being a fiercely competitive market, operates in an environment where, broadly, the price is dictated by the customer.

Historically, the industry operates on the basis following commercial basis:

Cost + profit = price

Designers will produce a design based on the client’s requirements; quantity surveyors price the design based on market testing, historical experience and experience (all of which contain unstated risk allowances). All too often the client is not willing to pay the price and through competition, where the suppliers cut their margins, alternative (more often than not inferior) specifications are adopted and scope (functionality) is reduced to arrive at the price that the client is willing to pay.

The application of VM techniques from the outset of the project will establish the functionality required by the client and the necessary margins to sustain a profitable business for all the suppliers involved. If the resulting cost, using the principle:

Price – profit = cost

is lower than the estimate, the functional cost model will identify the areas of discrepancy. The client then will be in a position to work with the designers and suppliers to align cost with their required functionality without creating adversarial or confrontational circumstances.
3.12 FUNCTION PERFORMANCE SPECIFICATION (FPS)

A specification is a document that describes a designer’s or a user’s intentions to a provider of the item being specified. Commonly produced specifications can be broadly categorised in 3 types:

1. Absolute specifications – here the specifier describes exactly what is required to the nth detail and scope for innovation or alternative options are non-existent.

2. Performance specifications describe the output required from a component or subsystem, but does not indicate or dictate the means of delivering that output. For example, the specification may require the internal environment in a room to be 21 ±2 °C throughout the year, with occupancy of up to 50 people. There are a variety of air conditioning and heating systems that could achieve this and the choice is left to the provider. This type of specification provides scope for innovation and genuine competitive bidding.

3. Function performance specifications (FPSs) take this one step further where the specifier defines user requirements by what the item must do rather than what it must be. In the previous example, the specification might read 'the room shall facilitate comfortable gatherings for the users at all times' - where the function of the room is to enable meetings to take place - the definition of "comfortable" is flexible and could be negotiated with the users. Similarly, the number of people attending can also vary. This gives the provider ultimate flexibility and ability to devise a very innovative and competitive solution.

The FPS comprises a thorough functional description of the system or subsystem being specified as it relates to the user (a user-related function - URF). To each function it attaches measurable attributes (known as Compliance Criteria - CC). Each CC is assigned a target level which can be quantitative or qualitative. Each target level is provided with a degree of flexibility by defining tolerance bands.

The method is to undertake a function analysis of user’s needs, identify measurable CCs, and agree the target levels and acceptable degrees of freedom. Having completed the FPS they can explore trade-offs between the CC for different functions. This enables them to arrive at the most competitive offering that complies with the specification.
CHAPTER 4

THE NEED FOR VM IN CONSTRUCTION PROJECTS
(A.MUKHERJEE, S.AKRAM, P.NOWAK)

Construction projects, particularly for the last 6 or 7 decades, have been subjected to frequent reviews and reports. Interestingly enough, most of the conclusions of these reports carried a similar theme highlighting some basic issues with the nature and perspectives of the industry.

Back in 1944, the Simon Report\textsuperscript{22} identified 4 key root causes of inefficiency and difficulties in construction projects. These were:

1. Insufficient preparation by the client of particulars of the work to be carried out at pre-construction stage;
2. Extensive variations after the contract has been placed;
3. Indiscriminate procurement of suppliers based on cheapest quote;
   and,
4. Indefinite relationships between the main contractor and the nominated subcontractors.

A significant number of subsequent reports, including the Emerson report\textsuperscript{23} (1962), the Banwell report\textsuperscript{24} (1963), the Wood report\textsuperscript{25} (1975), Latham reports\textsuperscript{26} (1993, 1994), and the Egan report\textsuperscript{27} (1998) proposed recurring themes of changes needed in areas including:

1. The adversarial nature of operation and contractual relationships;

\textsuperscript{22} Simon Report, 1944
\textsuperscript{23} Emerson Report, 1962
\textsuperscript{24} Banwell Report 1963
\textsuperscript{25} Woods Report 1975
\textsuperscript{26} Latham Trust & Money 1993, Constructing the Team 1994
\textsuperscript{27} Egan Report 1998
2. Integration of supply chain;
3. Simplicity in contractual language and arrangements; and,
4. Moving away from the cheapest option towards the ‘best value’ option.

From this it is evident that the principles and methodology of VM are not only necessary in construction projects in order to achieve better value from the resources, but also to deliver its ancillary benefits such as better communication, improved team-working, better alignment of the supply chain and, last but not least, a better understanding of the project objectives.

The three central themes of any construction project (or indeed of any project) are cost (referring to the total cost of the project), time (referring to the project time scale, from inception to occupation) and quality (relating to the quality of the output). The challenge for the project team is to balance these three elements to deliver the project, with the output being compatible with the project specifications as deemed necessary by the party commissioning the project. From this perspective, VM techniques can be used at different stages of a construction projects to generate different requirements at those stages. This aspect has been further discussed in chapter 7.

However, it is necessary to recognise that construction projects typically can vary from a small value (say £100,000) to much larger values and complexities.

The application of VM in construction projects will predominantly depend on the value of a particular project and the level of the risks involved\textsuperscript{28}. Fig. 4.1. below indicates a typical risk & value (value here refers to the value of the project to the party commissioning it) matrix which can be used to identify the need and necessity of VM.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{risk_value_matrix.png}
\caption{Typical Risk & Value Matrix}
\end{figure}

\textsuperscript{28} For further information in this area see Value Management in Construction: A Client's Guide, CIRIA SP 129
For strategic – critical projects (high risk, high value), a full VM process is almost always justified. For strategic – security projects (high risk, low value) and tactical – profit (low risk, high value) projects, the necessity of VM exercise will have to be decided upon the merits of each project. For tactical – acquisition projects (low risk, low value) a VM exercise is not usually necessary.

The primary benefits of a VM exercise are:

1. A clear definition of what the owners and end users mean by value, thus providing a precise basis for making decisions throughout the project, on the basis of value. It also provides a tool for optimising the balance between differing stakeholders needs and expectations.
2. A basis for creating a clear project brief that reflects the project sponsor’s priorities and expectations, expressed on the basis of value and function. This improves communication between all the stakeholders so that each of them can understand and respect other’s constraints, expectations and requirements.
3. A basis for ensuring that the project is the most effective way of

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29 Based on the Kraljic matrix for supplier positioning
delivering the business benefits and satisfying the business needs. It also provides a functional basis for embellishing and refining the business case for the project, by addressing both the monetary and non-monetary benefits.

4. A functional basis for design development and management, through improved communications, mutual learning and enhanced collaborative working, thus leading to better technical solutions with enhanced performance and quality often through innovative solutions.

5. A functional mechanism to measure value, taking into account the monetary and non-monetary benefits and thus demonstrating value for money.

VM can be a very low cost, high benefit exercise. When integrated into the project management methodology early in the project life cycle, the cost may become almost negligible, due to the reduced need for subsequent reviews and opportunities for substituting VM for some of the routine appraisals and quality audits that are always necessary.

For projects, the benefits of a VM review are often perceived in terms of improved quality and reduced cost. However the “invisible” benefits can be just as, or more valuable. Consensus and mutual understanding between stakeholders, clear objectives, reduced risk of changes in scope and improved communications will help to ensure that the project meets the objectives of the client and is delivered within the relevant parameters.

The VM process has now been fully integrated in the PPP (Public Private Partnership) projects, where it is a pre-requisite for the process to demonstrate a successful VfM exercise.\(^{30}\)

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\(^{30}\) For further information regarding PPP projects and VfM requirements see EU – CLOMEC II forthcoming publications on PPP
CHAPTER 5

ACHIEVING VM IN CONSTRUCTION PROJECTS
(A.MUKHERJEE, S.AKRAM, P.NOWAK)

VM techniques are commonly used in construction projects to serve two contextual situations. They are either used to achieve resource savings when a project has potentially crossed its limiting values, which may be in terms of cost or time, or to create an interface platform between the project commissioning team and the project delivery team to facilitate the development of a project delivery plan in line with the expected project deliverables within the limits of the project resources.

VM sets of techniques are primarily founded on three principal themes:

- Achievement of tasks through involvement and teamwork, based on the premise that a team will almost always perform better than an individual;
- Using subjective judgement, which may or may not incorporate risk assessment; and,
- Value is a function of whole life cost and utility in its broadest sense.

The key decisions in the application of VM in projects relate to:

- When should it be used?
- Who should be involved?
- Who should perform the role of the facilitator?

A balance must be struck between early application before an adequate understanding of the problem and constraints have been achieved and late application, when conclusions have been drawn and opinions hardened. Although feasibility (when identifying suitable options) and pre-construction (before the design – freeze) would in most cases be suitable, each project should be examined on its merits.
The facilitator’s role is; to gain commitment and motivate participants; draw out all views and ensure a fair hearing; select champions to take forward ideas generated; and, to keep to the agenda. To achieve these goals the facilitator must be independent, possess well developed interpersonal and communication skills and be able to empathise with all participants. Although the facilitator needs to understand the nature of the project, this need not be at a detailed level. Large, complex or otherwise difficult projects may warrant an external specialist facilitator. The facilitator’s role is crucial to the success of the exercise and care needs to be taken over selection. Table 5.1. below identifies some of the pros and cons of using an internal or an external facilitator.

Table 5.1. Arguments for and against internal and external facilitators

<table>
<thead>
<tr>
<th>An internal facilitator</th>
<th>An external facilitator</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>For</strong></td>
<td><strong>Against</strong></td>
</tr>
<tr>
<td>• Familiarity with the project – may help the team to achieve a deeper understanding of the project</td>
<td>• May not be able to appraise their own work critically</td>
</tr>
<tr>
<td>• can contribute towards effective team building</td>
<td>• may not be able to introduce fresh and new ideas</td>
</tr>
<tr>
<td>• Fresh ideas are generated</td>
<td>• are more likely to confirm that their original approach is the most effective one</td>
</tr>
<tr>
<td>• items can be critically appraised without having to defend existing ideas or approaches</td>
<td>• Potential conflict between external and internal parties</td>
</tr>
<tr>
<td>• can bring experience from other more diverse projects and organisations</td>
<td>• the project team may be unwilling to implement recommendations made externally</td>
</tr>
<tr>
<td></td>
<td>• the ownership and liability for in particular design ideas may be unclear</td>
</tr>
<tr>
<td></td>
<td>• may experience a more difficult learning curve</td>
</tr>
</tbody>
</table>

5.1 OVERCOMING POTENTIAL DIFFICULTIES

Generally, the participants in a VM process are selected on the basis of their technical or managerial expertise, rather than their skills in contributing to a collaborative effort.

One of the most difficult areas for a VM process is, perhaps, to cope with the difficulties that may arise during the process. These can arise for many reasons – organisational, technical, personal or even due to the process itself. Some of the potential difficulties are listed below:

- A lack of belief in the process;
- Difficult people (with difficult personality traits) present;
- Members of the team, representing different organisations, may be working to different agendas which are not aligned to those of the VM process;
- Conflicts between individuals and a tendency to point scoring;
- Lack of effective communication leading to misunderstanding;
- Differing objectives for individuals and organisations;
- Blame culture;
- Attitudinal issues;
- Differing and perhaps unrealistic expectations; and,
- Individual insecurity – needing to make their mark.

However, it is important to remember that not all the conflicts are counterproductive. One of the keys to a successful VM process (in particular the VM workshops) is to select a multidisciplinary team with different backgrounds and attitudes. Apart from ensuring that all the necessary technical expertise is present, this is likely to introduce differing views and solutions to the problem being studied. One of the challenges for the process leader (or the facilitator) is to harness this conflict constructively, leading to innovative thinking and improved outcomes.

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Some of the difficult personality traits can be “the spectator” – does not easily participate; “the windbag” – tends to try to dominate discussions; “the rambler” – loses track and uses far-fetched examples; “the squatter” – will not change from initial position.
The general rules to achieve conflict resolution in group situations are equally applicable in these instances:

- Emotions and people issues should be separated – consideration must be given to each issue from both sides’ perspective.
- It is important to be even-handed, and not favour one party over another. Efforts should be made to understand why the emotions are being felt and to get the person feeling those emotions to express his/her feelings clearly.
- Focus should be on people’s interests and not their positions. Positions are normally taken as a way of avoiding talking about the problem. A list may be made of each protagonist’s interests (not their positions) on two separate sheets of paper and then the common points of interest can be identified, not focusing on the differences. It is likely that the points of common interest far outweigh the differences.
- The facilitator (or the process leader) should focus the group on exploring a variety of options so that each party gains something (a win – win scenario), avoiding any premature judgement or a single answer. There is usually more than one way to solve a problem. By encouraging each protagonist to consider the other person’s problem as well as their own can often help the individuals to understand the issues better and allow them to accommodate at least part of the other person’s requirements.
- When closing the process to reach an agreement, the facilitator should use objective criteria, rather than subjective parameters that are drawn directly from the problem. Use should be made of norms, standards and benchmarks, market conditions, precedence and published data. If one party’s demands are seen to be way out of line with the common practice, it is more likely that they may relax their demands.

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33 For further information on this see Barton R (1995) Achieving Participation in Large Groups, University of Canberra

43
CHAPTER 6

ETHOS FOR VM
(A.MUKHERJEE, S.AKRAM, P.NOWAK)

The issues and challenges involved in successfully delivering a construction project are often quite complex and wide ranging, with various degrees of risks attached. This invariably necessitates a significant number of decisions by those who are involved with the project, both in individual and team capacities. Poor or wrong decisions can be made under the competing pressures of time, budget and quality. As a result, all projects are likely to include unnecessary costs. There is a range of tools and techniques that can be utilised to maximise VM efforts which should enable the team to minimise these unnecessary costs - these include:

- Creative approach to problem solving;
- A systematic, staged approach – with a clear definition of objectives and scope at the outset - focus on the customer requirements, consideration of the organisational environment - internal and external influences, multi-disciplinary team effort - positive human dynamics;
- Function analysis;
- A workshop format with a structured job plan that separates creativity from evaluation and development;
- Effective use of methods and tools - independent facilitation; and,
- Continuity in VM.
CHAPTER 7

VM AT DIFFERENT STAGES OF CONSTRUCTION PROJECTS
(A.MUKHERJEE, S.AKRAM, P.NOWAK)

The timing of V/M interventions in the development of a construction project depends upon the particular circumstances of the project. Typical stages within project development where V/M techniques can be applied are:

- Inception - VM
- Feasibility - VM
- Strategy - VM
- Preconstruction - VM/VE
- Construction - VE
- Services Commissioning - VE, VA
- Completion, hand over & occupation - VA
- Post completion review/ project close out - VA

An example of utilisation of VM at key stages in a project framework is shown in fig. 7.1.

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34 For clarification regarding the project stages see the Code of Practice for Project Management for Construction and Development (4th ed. 2010)
Fig. 7.1. VM at key stages of a construction project\textsuperscript{35}

\textsuperscript{35} Source: Code of Practice for Project Management (CIOB) 4\textsuperscript{th} Ed.
CHAPTER 8

CASE STUDIES\(^\text{36}\)
(A.MUKHERJEE, S.AKRAM, P.NOWAK)

8.1 CASE STUDY 1

Housing improvement projects: This case study illustrates how utilisation of VM within a partnering framework led to continually improved production and cost efficiency without compromising supply chain profit margins.

8.2 PROJECT BACKGROUND

The £144m project involved improvement of homes across South Lanarkshire by providing new kitchens and bathrooms, new electrical rewiring, external envelope renewal, environmental improvements and consequential projects such as access improvements in multi-storey buildings (lift replacements and so on).

Previous projects of a similar nature had been procured traditionally and resulted in a confrontational culture, poor quality outputs and low productivity.

The challenge was to achieve end-user satisfaction whilst still maintaining cost and time commitments.

8.3 UTILISATION OF VM

- A Partnering Charter was developed between the public and the private sector participants – linking all parties through a quality system allowed them to exchange information and work together and also improved the communication between the client and supply chain.
- Decisions based on value – opportunities for savings of £9.75m were identified across the supply chain over the life of the project.
- Combining VM with other collaborative strategies – all parties worked together to resolve issues using value as the key decision criterion.
- “Right first time” policy – VM principles emphasise the need to strive for the highest levels of customer satisfaction. Every small defect, delay or weakness in communication would have an adverse effect on service satisfaction. Through VM workshops, specific performance improvement targets were agreed and incorporated which were continually monitored against pre-agreed KPI's.
- Step-changes in production – VM experience shows that improvement comes from many small steps at operational level. This is encouraged and facilitated by customer satisfaction and management initiatives. Formal VM workshops kick-started all the small steps.
- Tracking improvements – the management team also met between the workshops to discuss actual and potential improvements – these meetings ensured an up-to-date record of improvements. Formal VM studies were also carried out to build upon the initial success of the initial VM process and to set an agenda to explore ways to further improve value for money.
8.4 LESSONS LEARNED

- Independent objective facilitation enabled the team to maintain progress and adopt new approaches;
- VM has resulted in more effective working, significantly improved customer satisfaction and reduced costs;
- VM contributed to improvement of clearer brief and decision taking;
- VM enhanced the value and benefits for the end users;
- the VM process, by reducing cost and offering savings, improved affordability and value for money;
- the VM process helped to improve productivity, efficiency, collaboration and trust;
- VM helped to reduce wastes and defects;
- VM also facilitated earlier management involvement thus helping in potential issue identification and resolution; and,
- Overall, the adaptation of VM resulted in better realisation of project benefits where previous methods had failed.

8.5 CASE STUDY 2

This case study demonstrates a successful utilisation of VM for a £17m new build project where cost savings of £1.3m were achieved at a VM study cost of £120,000 – a benefit to cost ratio of more than 10:1.

The partnership approach was adopted when previous attempts to design, tender, procure and construct using traditional procurement routes failed to meet capital budget constraints and satisfy user requirements.

The design and construct partnership was appointed on best-value principles (not simply awarding on lowest price, but evaluating commitment to the client's needs and budget).

The proposed building was constrained by existing buildings, car parking and natural features – including a large tree and a pond which was home to a colony
of great crested newts. The failure to gain approval for previous proposals had engendered doubts that a properly functioning building could be delivered within budget, on time and also satisfy the aspirations of user groups.

The challenge to the project team was to deliver the new building which; must not exceed the capped capital budget; would be ready for occupation by the stated moving-in date; would meet user requirements; and, would engage and involve all the stakeholders, in particular the users of the building and those responsible for its upkeep.

From the outset, the project team's commitment to a programme of VM and other full-team workshops led to a successful project delivered on time and to budget, with high levels of client and user satisfaction. The project team achieved this by identifying and enhancing value, developing a common understanding of user needs and defining common objectives.

8.6 UTILISATION OF VM

- Core Group – a Core Group was established which comprised a representative from each partner organisation. It met monthly to maintain an overview of the process and to support decision making;
- Trust – the increase in positive experiences, created by the successful series of VM workshops significantly raised the level of trust within the team; and,
- Early involvement - involvement of end users ensured that users’ needs were identified before value-adding proposals were generated, which resulted in evaluation of any potential for value enhancement before spending time on the detailed development of the proposals.

Each workshop featured a team exercise to reinforce a learning point or introduce a process.

The Core Group was positively committed to empowering the team members to resolve issues before they became problems. This led to more timely issue
resolution, maintenance of the project and a lower cost of resolving issues, as senior management had less input on relatively minor matters.

- Knowledge-sharing – one of the principals of VM is that added value is driven not only by workshops but also by the sharing of explicit and tacit knowledge throughout the project and then using that knowledge to benefit the project. During review workshops, all team members benefited from communicating their views on project and team successes and opportunities. Discussions were held, both formally and informally, about how these might impact on the project, learning in cross-organisational groups how to use this information to benefit the scheme.

- Mitigating risk - learning from the previous failed attempts to procure the new buildings by traditional means had not yielded proposals that could be delivered within time and budget constraints and to the satisfaction of the end users, the VM programme was implemented to help to reduce uncertainties.

- Reducing budget uncertainty – as the project developed, it became clear that there was a potential mismatch between resources and expectations. This caused budget uncertainty. The programme of face-to-face full team VM workshops brought these issues into the open, enabling the team to propose and develop creative solutions to these problems. Further workshops, during the design, development and procurement stages provided more opportunities for both costs savings and enhancements to meet user needs. The outcome of these workshops was a reduction in construction costs of 20% whilst retaining essential client and end user functionality.

- Cost-effectiveness of VM process – the overall cost of conducting the VM programme, including the direct and indirect costs, was approximately £120,000. The realised cost savings were £1.3m, representing a payback of over 10:1.

8.7 LESSONS LEARNED

- The success of the VM workshop programme has led the client to adopt the same approach on subsequent projects;
• the process allowed the client to feel part of the project and also involved the end users, enabling both client and user needs to be met in a resourceful and value-adding manner;
• The workshops helped the team to push the boundaries of cost, time and quality. Normally, one would perhaps expect a gain in one of these criteria at the expense of the other two, but in this project the team achieved better-than-expected performance in all three criteria;
• Overall, the VM process improved the definition and articulation of value in the context of the project;
• VM contributed to improvement of clearer brief and decision taking;
• VM enhanced the value and benefits for the end users;
• the VM process, by reducing cost and offering savings, improved affordability and value for money;
• the VM process helped to improve productivity, efficiency, collaboration and trust;
• VM helped to reduce waste and defects;
• VM also facilitated earlier management involvement thus helping in potential issue identification and resolution; and,
• Overall, the adaptation of VM resulted in better realisation of project benefits where previous methods had failed.
Project photographs

Hextable Dance Project,
Kent

Open University Library,
Milton Keynes
(Case Study 2)

Kintry Housing
Partnership, Scotland
Home Happening, South Lanarkshire
(Case Study 1)

NHS Teaching Hospital, North Staffordshire

Withington Community Hospital
Manchester
CHAPTER 9

REFERENCES AND FURTHER INFORMATION

PART II

INTEGRATED VALUE AND RISK MANAGEMENT FOR REAL ESTATE PROJECTS IN THE PRE-DECISION PHASE
CHAPTER 10

INTRODUCTION
(A.MINASOWICZ)

Analysis of construction risk associated with an investment project can be analysed using qualitative and quantitative methods. In the first part of this textbook, we focus on the philosophy of an integrated approach to management of value and risk, as well as qualitative analyses.

In the second part, we presented the selected quantitative procedures of analysis of construction value and risk of an investment project. These procedures apply linguistic assessment to switch to quantitative risk analysis for planned projects. The procedures are solved using the fuzzy sets theory. The procedures are constructed so that they do not require the user’s background in the fuzzy set theory and they are easy to apply.

This part contains the following procedures that solve the selected problems in the field of value and risk management associated with investment decision-making:

• Analysis of a construction project in the pre-decision phase, taking into account the economic trends and the fuzzy selection criterion. This procedure allows for selection of the investment project, which corresponds best with the investor’s risk preferences, at the strategic risk building stage.
• Analysis of selection of the investment project based on effectiveness of the project to be implemented, presented in fuzzy form. This procedure can be applied at the feasibility study stage during the preliminary risk assessment to examine the susceptibility of NPV to modification of design and construction solutions.
• Assessment of the investment project at the stage of a feasibility study with regard to risk and probability of reaching of the NPV value. This procedure enables assessment of the NPV value and its probability for
the investment project analysed at a given stage, or the project risk review and preparation of risk reduction guidelines

- Analysis of a multi-criteria selection of the best offer in tender procedures using expert knowledge based on fuzzy premises. This procedure may be applied at the stage of integrated value and risk management upon selection of the bidder during tenders.
- The time-cost analysis of the construction project planned, taking into account the risk based on expert knowledge using fuzzy sets. The procedure can be applied at the level of review of risk for a specific project to define the scenarios (optimistic, pessimistic) of implementation of the project, taking into account the specific time and cost.
CHAPTER 11

ANALYSIS OF A CONSTRUCTION PROJECT IN THE PRE-DECISION PHASE, TAKING INTO ACCOUNT ECONOMIC TRENDS AND FUZZY SELECTION CRITERION
(A.MINASOWICZ)

11.1 ABSTRACT

This analysis provides the possibility to select the best projects, taking into account the company strategic risk policy. Having specified the net present value, we conduct simulations for various economic scenarios. In this way, we obtain the estimated net present value as a random variable for many potential projects. Selection of the optimum project is made on the basis of the potential benefits, defined on the basis of the expected net present value and on the basis of the potential risk, defined by the coefficient of variability of the net present value. Depending on selection of one of three strategies (conservative, balanced or aggressive) by the investor, analysis on the basis of fuzzy sets theory is applied. The procedure proposed allows for selection of investment projects that match best with the investor’s strategy at the stage of assessment of strategic risk.
11.2 INTRODUCTION

Investing can be referred to as giving up certain current benefits on behalf of uncertain benefits in the future. Most often, these benefits consist of capital in the form of money. Engaging this capital, we assume a high probability of earning profits in the future, or increasing the capital amount, which usually constitutes the investor’s expected benefit. As is well known, this probability of achieving the expected benefits is never equal to 1. Most often, investment projects are subject to risks. The investor is never 100% sure that an investment will bring the specific profit as assumed.

To enable the investor to make the best use of their money, that is, to provide the highest possible profit from the investment selected at a minimum risk, various methods of determining effectiveness of investment projects are employed. One of these is the NPV method.

In this chapter, we propose the application of fuzzy investment selection criterion as a development of the already used method of assessment of effectiveness using the variation coefficient.

First of all, we analyse selection of the investment project, for which the $E(NPV)$ has been established - the expected net present value depending on the economic scenario. In this way, the random variable $NPV_i$ will be characterized by such statistical quantities as:

- Expected value
- Standard deviation
- Variation coefficient

11.3 DESCRIPTION OF THE METHOD

The example presents an analysis of four variants of the investment strategy – projects A, B, C and D. The possible net present values (NPV) of these projects are determined by the five possible trends of macro- and meso-economic changes in the environment of these projects. At the same time, stabilization of
microeconomic factors associated with the company strategy and the project itself is expected.

Experts have determined the probability of occurrence of these economic trends. Thus, five different NPV values are possible, and a certain probability is assigned to each of these. As a result, five investment scenarios have been devised for the four projects analysed, for which the expected $E(NPV)$ values, their standard deviation $\sigma(NPV)$ and variation coefficients $C(NPV)$ have been established.

**Table 11.1. A breakdown of $E(NPV)$ values for 4 projects depending on the economic scenario**

<table>
<thead>
<tr>
<th>Economic trends</th>
<th>Probabilities</th>
<th>Project A (NPV)</th>
<th>Project B (NPV)</th>
<th>Project C (NPV)</th>
<th>Project D (NPV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very good (1)</td>
<td>0.1</td>
<td>200</td>
<td>100</td>
<td>250</td>
<td>210</td>
</tr>
<tr>
<td>Good (2)</td>
<td>0.2</td>
<td>110</td>
<td>55</td>
<td>140</td>
<td>150</td>
</tr>
<tr>
<td>Average (3)</td>
<td>0.3</td>
<td>25</td>
<td>25</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td>Difficult (4)</td>
<td>0.3</td>
<td>-20</td>
<td>15</td>
<td>-40</td>
<td>-60</td>
</tr>
<tr>
<td>Bad (5)</td>
<td>0.1</td>
<td>-100</td>
<td>-50</td>
<td>-165</td>
<td>-200</td>
</tr>
<tr>
<td>$E(NPV)$</td>
<td></td>
<td>33.5</td>
<td>28.00</td>
<td>33.5</td>
<td>28.00</td>
</tr>
<tr>
<td>$\sigma(NPV)$</td>
<td></td>
<td>81.27</td>
<td>36.41</td>
<td>111.89</td>
<td>118.14</td>
</tr>
<tr>
<td>$C(NPV)$</td>
<td></td>
<td>2.43</td>
<td>1.30</td>
<td>3.34</td>
<td>4.22</td>
</tr>
</tbody>
</table>

Calculations have been presented for project A as an example.

$E(NPV)_A = 0.1 \cdot 200 + 0.2 \cdot 110 + 0.3 \cdot 25 + 0.3 \cdot (-20) + 0.1 \cdot (-100) = 20 + 22 + 7.5 + (-6) + (-10) = 33.5$

$\sigma^2(NPV)_A = 0.1 \cdot (200 - 33.5)^2 + 0.2 \cdot (110 - 33.5)^2 + 0.3 \cdot (25 - 33.5)^2 + 0.3 \cdot (-20 - 33.5)^2 + 0.1 \cdot (-100 - 33.5)^2 = 0.1 \cdot 166.5^2 + 0.2 \cdot 76.5^2 + 0.3 \cdot (-8.5)^2 + 0.3 \cdot (-53.5)^2 + 0.1 \cdot (-133.5)^2 = 2772.22 + 1170.45 + 21.67 + 858.67 + 1782.22 = 6605.23$

$\sigma(NPV)_A = 81.27$

$C(NPV)_A = 81.27 : 33.5 = 2.43$
Fig. 11.1. Chart of correlation $\sigma$(NPV)/$E$(NPV) = $C$(NPV) for each project

Source: own work

On the basis of the chart, it can be concluded that investment A is better than investment C, since at the same quantity of profit expected, the direct risk, represented by standard deviation, is lower. An analogous situation can be observed with regard to investment projects B and D. In this pair, investment B is more beneficial. The question is, how to compare investment projects A and B.

On one hand, investment A provides a higher expected profit; on the other hand, the expected risk is greater. A comparison of variation coefficients will not provide a measurable result. We should ask about the investor’s objectives. Does the investor want investment, which can bring a substantial profit, but which is also associated with a risk of loss; or, perhaps, is the investor more conservative, focused on investment projects, which are characterised by low risk and lower profits. In other words, what is the investor’s attitude towards risk?

The study proposes an innovative approach to analysing a selection of an investment project on the basis of a fuzzy criterion of selection. Let us assume
that 20 potential projects are to be analysed, and the potential investor is interested in all of them. Each investment project can be characterized by such values as: the expected value $E(\text{NPV})$, standard deviation $\sigma(\text{NPV})$ and variation coefficient $C(\text{NPV})$. These values have been determined in precisely the same manner as in the previous example.

Table 11.2. Values $E(\text{NPV})$, $\sigma(\text{NPV})$, $C(\text{NPV})$ for each project
Source: own work

<table>
<thead>
<tr>
<th>i</th>
<th>$E(\text{NPV})_i$</th>
<th>$C(\text{NPV})_i$</th>
<th>$\sigma(\text{NPV})_i$</th>
<th>$X_i$</th>
<th>$Y_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>75.45</td>
<td>0.94</td>
<td>70.58</td>
<td>0.79</td>
<td>0.84</td>
</tr>
<tr>
<td>2</td>
<td>84.31</td>
<td>1.09</td>
<td>91.76</td>
<td>0.90</td>
<td>1.00</td>
</tr>
<tr>
<td>3</td>
<td>15.74</td>
<td>0.22</td>
<td>3.42</td>
<td>0.05</td>
<td>0.09</td>
</tr>
<tr>
<td>4</td>
<td>21.40</td>
<td>0.38</td>
<td>8.11</td>
<td>0.12</td>
<td>0.26</td>
</tr>
<tr>
<td>5</td>
<td>11.83</td>
<td>0.13</td>
<td>1.49</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>6</td>
<td>49.99</td>
<td>0.55</td>
<td>27.66</td>
<td>0.47</td>
<td>0.44</td>
</tr>
<tr>
<td>7</td>
<td>46.34</td>
<td>0.29</td>
<td>13.35</td>
<td>0.43</td>
<td>0.17</td>
</tr>
<tr>
<td>8</td>
<td>60.98</td>
<td>0.43</td>
<td>25.98</td>
<td>0.61</td>
<td>0.31</td>
</tr>
<tr>
<td>9</td>
<td>53.71</td>
<td>0.94</td>
<td>50.29</td>
<td>0.52</td>
<td>0.84</td>
</tr>
<tr>
<td>10</td>
<td>34.81</td>
<td>0.53</td>
<td>18.61</td>
<td>0.28</td>
<td>0.42</td>
</tr>
<tr>
<td>11</td>
<td>74.33</td>
<td>0.40</td>
<td>29.41</td>
<td>0.77</td>
<td>0.28</td>
</tr>
<tr>
<td>12</td>
<td>25.42</td>
<td>0.19</td>
<td>4.90</td>
<td>0.17</td>
<td>0.07</td>
</tr>
<tr>
<td>13</td>
<td>31.97</td>
<td>0.63</td>
<td>20.17</td>
<td>0.25</td>
<td>0.52</td>
</tr>
<tr>
<td>14</td>
<td>92.79</td>
<td>0.85</td>
<td>79.22</td>
<td>1.00</td>
<td>0.76</td>
</tr>
<tr>
<td>15</td>
<td>74.50</td>
<td>0.68</td>
<td>50.60</td>
<td>0.77</td>
<td>0.57</td>
</tr>
<tr>
<td>16</td>
<td>48.10</td>
<td>0.91</td>
<td>43.87</td>
<td>0.45</td>
<td>0.82</td>
</tr>
<tr>
<td>17</td>
<td>63.68</td>
<td>0.64</td>
<td>40.70</td>
<td>0.64</td>
<td>0.53</td>
</tr>
<tr>
<td>18</td>
<td>54.59</td>
<td>0.31</td>
<td>17.13</td>
<td>0.53</td>
<td>0.20</td>
</tr>
<tr>
<td>19</td>
<td>66.88</td>
<td>0.29</td>
<td>19.36</td>
<td>0.68</td>
<td>0.17</td>
</tr>
<tr>
<td>20</td>
<td>69.03</td>
<td>0.54</td>
<td>37.10</td>
<td>0.71</td>
<td>0.43</td>
</tr>
</tbody>
</table>

In this table, the columns have the following meaning:

$E(\text{NPV})_i$ – expected net present value of i-th project

$C(\text{NPV})_i$ – variation coefficient of i-th project

$\sigma(\text{NPV})_i$ – standard deviation of i-th project

Each of these quantities has its maximum and minimum value for the 20 projects analysed.
Table 11.3. Extreme values E(NPV), C(NPV), σ(NPV)
Source: own work

<table>
<thead>
<tr>
<th></th>
<th>E(NPV)</th>
<th>C(NPV)</th>
<th>σ(NPV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>min</td>
<td>11.83</td>
<td>0.13</td>
<td>1.49</td>
</tr>
<tr>
<td>max</td>
<td>92.79</td>
<td>1.09</td>
<td>91.76</td>
</tr>
<tr>
<td>Δ</td>
<td>80.96</td>
<td>0.96</td>
<td>90.27</td>
</tr>
</tbody>
</table>

Min – minimum value
Max – maximum value
Δ = max-min

For the purposes of the example presented, two columns have been added in table 2-2.

$X_i$ - relative value of the expected net present value $E(\text{NPV})_i$,
$Y_i$ - relative value of the variation coefficient $C(\text{NPV})_i$,

where:

$$X_i = \frac{E(\text{NPV})_i - \min(E(\text{NPV})_i)}{\Delta_{E(\text{NPV})}} = \frac{E(\text{NPV})_i - \min(E(\text{NPV})_i)}{\max(E(\text{NPV})_i) - \min(E(\text{NPV})_i)}$$

(11.1)

$$Y_i = \frac{C_i - \min(C_i)}{\Delta_C} = \frac{C_i - \min(C_i)}{\max(C_i) - \min(C_i)}.$$  

(11.2)
We now need to define the investment projects in terms of their attractiveness. Attractiveness is understood as the investor’s “willingness to invest” in a given project. The criteria of investment attractiveness – or willingness to invest in a given project – are values X and Y where:

1. \( X \) – is the relative net updated value – information on the expected profits;
2. \( Y \) – is the relative variation coefficient - information on risk.

Variables X and Y have been presented as linguistic variables.

A linguistic variable is an input or an output quantity or a status variable, which is described by linguistic scores, known as linguistic values. Linguistic values appear in models along with linguistic variables, subject to e.g.:

- High pressure;
• Strong water jet;
• Young age of a person;
• True information.

We distinguish the following variables with membership function:
• Continuous;
• Discrete.

The linguistic space of a variable is a set of all linguistic values used to assess a given linguistic variable. In the case of X and Y values, the linguistic space is presented as in the chart below.

Fig. 11.3. Graphic interpretation of linguistic space X or Y (membership function in continuous format)
Source: own work

For the sake of simplification, value of membership level < 0,5 was eliminated.
Another simplification is the presentation of linguistic space of variables X and Y in discrete format. It is based on presentation of the membership function as the appropriate membership levels – membership function values in appropriate points.

Fig. 11.4. Graphic interpretation of linguistic space X or Y (membership function in continuous format), excluding value of membership level ≤0.5
Source: own work
To define precisely the investment strategy, two-argument function $A$ – investment attractiveness – was applied. It can be stated undoubtedly that as $X$ increases, so does the investment attractiveness; in the case of $Y$, it is quite the opposite. The higher the value of $Y$, the more risky is the investment. Thus its attractiveness, and the value of $A$, decreases.

The value of this function is defined by the investor by verbal formulation of values of $A(X,Y)$. This will be the so-called base of rules. The base of rules is the most significant part of the fuzzy model. The base of rules is the most
significant part of the model (regulator) containing information on its structure. It can be compared to a tent frame, on which the cloth is stretched. The shape of its frame is decisive for the shape and appearance of the entire tent. The node points are determined by coordinates X and Y of membership level 1. The investor determines the values in the node points of the frame. These values will be between 0 – absolute unwillingness to invest (minimum investment attractiveness) to 1 – absolute willingness to invest (maximum investment attractiveness), rounded up to one tenth (0;0.1;0.2,…,1).

The following base of rules was used to define the values of \( A(X,Y) \):

If \( X = \ldots \) and \( Y = \ldots \) Then \( A(X,Y) = \ldots \)

In the first place, the base of rules was presented in tabular format. Depending on the value of X and Y, the investor defines his or her willingness to invest. Values in the table are the so-called node points. Their credibility, or degree of membership, is equal to one and it corresponds with the values of individual linguistic variables, which also have the membership level of one. For linguistic variables X and Y, these are values (0 ; 0.25 ; 0.5 ; 0.75 ; 1). Spaces between node points are established by surfaces with peaks in the neighbouring node points. In the examined example, these are planes. Moreover, the base of rules \( A(X,Y) \) has been presented as a frame in the three-dimensional Cartesian system.

Investors have been divided into three groups:

- aggressive investors;
- balanced investors;
- conservative investors.

For each group, an exemplary base of rules was established.

Aggressive investors are characterised by great willingness to take risks, and thus they are focused on the greatest net present value possible. Risk associated with the investment is analysed as the second factor.
Table 11.5. Base of rules $A(X,Y)$ established by an aggressive investor
Source: own work

<table>
<thead>
<tr>
<th>Aggressive investor</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>min</td>
<td>low</td>
<td>average</td>
<td>high</td>
<td>max</td>
</tr>
<tr>
<td>Y</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
<td>0.7</td>
<td>1.0</td>
</tr>
<tr>
<td>0.25</td>
<td>0.3</td>
<td>0.3</td>
<td>0.4</td>
<td>0.6</td>
<td>0.9</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2</td>
<td>0.2</td>
<td>0.3</td>
<td>0.5</td>
<td>0.8</td>
</tr>
<tr>
<td>0.75</td>
<td>0.1</td>
<td>0.1</td>
<td>0.3</td>
<td>0.5</td>
<td>0.8</td>
</tr>
<tr>
<td>1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.2</td>
<td>0.4</td>
<td>0.7</td>
</tr>
</tbody>
</table>

The table is to be read as follows (an example)
If $X = 0$ and $Y = 0$ Then $A(X,Y) = 0.3$

Fig. 11.6. A graphic representation of the investment attractiveness function frame for an aggressive investor
Source: own work
In the drawing, the axes have been marked as follows:
- X - relative net value E(NPV);
- Y - relative variation coefficient;
- A(X,Y) - investment attractiveness.

The chart presents node points established in accordance with the values provided by the investor in the table. Connecting these with sections, we obtain a frame of the base of rules.

A balanced investor focuses both on the net present value and the risk associated with various economic scenarios.

**Table 11.6. Base of rules A(X,Y) defined by a balanced investor**
Source: own work

<table>
<thead>
<tr>
<th>Balanced investor</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>min</td>
</tr>
<tr>
<td></td>
<td>low</td>
</tr>
<tr>
<td></td>
<td>average</td>
</tr>
<tr>
<td></td>
<td>high</td>
</tr>
<tr>
<td></td>
<td>max</td>
</tr>
</tbody>
</table>
For conservative investors, certainty of profit is the most significant issue, and thus they first take into account the relative risk. The second factor is the profit value.
Table 11.7. Base of rules $A(X,Y)$ defined by a conservative investor
Source: own work

<table>
<thead>
<tr>
<th>Conservative investor</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Y</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>min</td>
</tr>
<tr>
<td>0.25</td>
<td>low</td>
</tr>
<tr>
<td>0.5</td>
<td>average</td>
</tr>
<tr>
<td>0.75</td>
<td>high</td>
</tr>
<tr>
<td>1</td>
<td>max</td>
</tr>
</tbody>
</table>

Fig. 11.8. A graphic representation of the investment attractiveness function frame for a conservative investor
Source: own work
Levels of membership in discrete format for function A, for all estimations of arguments X and Y, have been presented in table 2-8. These levels assume values equal to the arithmetic average of levels of membership of individual X and Y arguments. Only for node points, the membership level is equal to 1, and the level of membership of intermediate points is the lesser, the greater is their distance from node points.

Table 11.8. Tabular presentation of membership (credibility) levels of investment points
Source: own work
A breakdown of 20 investment projects and a frame of the base of rules for each type of investor has been presented in drawings 11.11, 11.12, 11.13. Node points are presented as “crosses”. Their value is shown as in fig. 11.9.

\[ A(X,Y) \]

**Fig. 11.9. The format of presentation of node points on the final drawing**
Source: own work

The chart presenting values X and Y for individual investment projects has been used as a background for the frame of the base of rules, depending on the investor’s willingness to take risks.

**Fig. 11.10. X and Y values of 20 investment projects**
Source: own work
Fig. 11.11. Values X and Y and A(X,Y) for an aggressive investor
Source: own work
Fig. 11.12. Values X and Y and A(X,Y) for a balanced investor
Source: own work
Fig. 11.13. Values X and Y and A(X,Y) for a conservative investor
Source: own work

11.4 CONCLUSIONS

In the case of an aggressive investor, the most attractive investment projects are 2 and 14, for a balanced investor – 11 and 19 will be the best. A conservative investor will be most interested in projects 3 and 12.

As has been mentioned, investing means the giving up of present means for the sake of future profits. These profits are always associated with a certain risk, for instance, due to the uncertainty of occurrence of economic scenarios. Every investor is willing to take some risk. To characterise the attitude towards risk and select the most appropriate – or, in other words, attractive - project for a given investor, we have proposed the introduction of a definition of attractiveness as a function of two arguments: profit and risk.

Profits are represented by the expected net present value $E(NPV)$, and the relative risk is represented by the variation coefficient of value $C(NPV)$. These quantities serve as criteria for selection of the investment project. These criteria show us the level of interest of the investor: from no interest (0) to maximum interest (value of 1). In this way, in the entire space of possible investment variants, some are more and others are less attractive. Moreover, it is the investor who makes the decision, which places are perceived as more attractive. Defining values $E(NPV)$ and $C(NPV)$ as linguistic values, the investor or contractor may comment verbally on attractiveness, establishing the so-called base of rules. This allows for devising of the so-called frame, which is a source of information on the entire scope of the investment project (from min value min = 0 to max value =1).

In the example presented, three typical models of investors have been provided:

- Aggressive investor;
- Balanced investor;
- Conservative investor.

As a result of presenting the projects on a chart $Y / X$ and applying the frame of values $A(X,Y)$, the most appropriate projects among the examined 20 were selected for each type of investor.
11.5 THE PROCEDURE ALGORITHM – A BLOCK DIAGRAM

The procedure described has been presented below in a block diagram format (Fig. 11.14).

Fig. 11.14. A block diagram of a construction project analysis at the pre-decision stage, taking into account the economic trends and using the fuzzy selection criterion.
Source: own work
CHAPTER 12

ANALYSIS OF SELECTION OF THE INVESTMENT PROJECT BASED ON EFFECTIVENESS OF THE PROJECT TO BE IMPLEMENTED, PRESENTED IN FUZZY FORM (A.MINASOWICZ)

12.1 ABSTRACT

In order to minimise the risk associated with a specific construction project, or for the purpose of final selection between several investment projects, an analysis based on historic data has been proposed. For this purpose, for a given investment project, it is necessary to specify the expected net present value as the potential profit from investment. The next step is gathering of historic data on the basis of completed projects of a profile similar to the one in question. Thanks to the trapezoidal fuzzy set representation, we obtain full information on the possible deviations from the net present value, taking into account both historic results and expert knowledge. Additionally, the analysis applies the so-called identity coefficient, which specifies the degree of similarity between the examined project and the completed historic projects. Having access to representation of deviations in a fuzzy set format, the investor has all information necessary on the possible deviations and is able to compare the possible technological and building variants of implementation of a construction
project to avoid the potential risk. The procedure proposed can be applied at the stage of a strategy and feasibility study during the preliminary risk analysis.

12.2 INTRODUCTION

The group of basic indicators of the investment project implementation process consists of the following parameters:
- The time of implementation of the construction project;
- The cost of implementation of the construction project;
- Profit earned through capital investment.

Unfortunately, these values, obtained during the project implementation, are changeable and sometimes they exceed the planned values. This may be due to factors that are independent or dependent on the project type. These factors may include:
- Underestimation of costs at the investment planning stage;
- Application of unverified equipment and/or technologies;
- Design and/or construction errors;
- Low quality of performance;
- Selection of unreliable suppliers and/or contractors for the project.

An error in any of these fields results in a risk of change of the investment assumptions. The investor should thus aim at minimization of these errors. This can be achieved, among other things, thanks to the following activities:
- Conduct a thorough technical and economic analysis of the investment project along with a precise estimation of costs;
- Establish a reserve for unexpected works in the investment project budget;
- Select a reliable and experienced contractor;
- Obtain a performance bond to secure the investor against losses due to; delay or inappropriate quality of implementation of the project.

Despite many steps undertaken to minimise the risk, some projects are more risky than others. Such a phenomenon is described in the chapter as susceptibility of the investment project.
Susceptibility is understood as sensitivity of the investment project or probability of changes in the investment costs/profits.

**12.3 DESCRIPTION OF THE METHOD**

In order to characterise the investment project, data from completed projects can be used, as well as expert opinions. On the basis of these, it is possible to establish the NPV of the project examined as a fuzzy value. To specify the NPV in fuzzy format, it is possible to use data from previously completed investment projects, as well as expert opinions.

The analyses include a coefficient referred to as the „identity coefficient”. It specifies the similarity between the project examined and projects taken into account for the purpose of cost estimation (the so-called “historic investment projects”). These coefficients are established by experts. The value of coefficients ranges from 0.5 to 1. The minimum value, or 0.5, means that the investment project completed, for which NPV is the source of comparison, is similar only in 50 per cent of the investment project examined. Let us assume that for each investment project, we use the opinions of three independent experts and seven most similar historic projects. The table below presents a breakdown of these values for a single investment project.
Table 12.1. A tabular breakdown of historic data and expert opinions
Source: own work

<table>
<thead>
<tr>
<th>i</th>
<th>( k_i )</th>
<th>wsp(_{E1})</th>
<th>wsp(_{E2})</th>
<th>wsp(_{E3})</th>
<th>wsp(_i)</th>
<th>( w_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8</td>
<td>0.9</td>
<td>0.8</td>
<td>0.9</td>
<td>0.87</td>
<td>0.104</td>
</tr>
<tr>
<td>2</td>
<td>0.75</td>
<td>0.8</td>
<td>0.75</td>
<td>0.75</td>
<td>0.77</td>
<td>0.092</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0.9</td>
<td>0.95</td>
<td>1</td>
<td>0.95</td>
<td>0.114</td>
</tr>
<tr>
<td>4</td>
<td>0.9</td>
<td>0.6</td>
<td>0.65</td>
<td>0.7</td>
<td>0.65</td>
<td>0.078</td>
</tr>
<tr>
<td>5</td>
<td>1.15</td>
<td>0.7</td>
<td>0.6</td>
<td>0.75</td>
<td>0.68</td>
<td>0.082</td>
</tr>
<tr>
<td>6</td>
<td>0.95</td>
<td>1</td>
<td>1</td>
<td>0.95</td>
<td>0.98</td>
<td>0.118</td>
</tr>
<tr>
<td>7</td>
<td>0.6</td>
<td>0.8</td>
<td>0.7</td>
<td>0.65</td>
<td>0.72</td>
<td>0.086</td>
</tr>
<tr>
<td>Exp. 1</td>
<td>8</td>
<td>0.75</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>0.108</td>
</tr>
<tr>
<td>Exp. 2</td>
<td>9</td>
<td>0.8</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>0.108</td>
</tr>
<tr>
<td>Exp. 3</td>
<td>10</td>
<td>0.85</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>0.108</td>
</tr>
</tbody>
</table>

|  | sum= | 8.32 | 1.00 |

NPV values are presented in the table in column \( (k_i) \). At the same time

\[
k_i = \frac{NPV_i^{\text{fin}}}{NPV_i^{\text{def}}}, \tag{12.1}
\]

where:

- \( k_i \) – the relative \( NPV_i \) value
- \( NPV_i^{\text{fin}} \) – the final \( NPV_i \) value after the defined time period from commencement of the project
- \( NPV_i^{\text{def}} \) – \( NPV_i \) value assumed for the project after the defined time period from commencement of the project

Table 12.1 presents 10 \( k_i \) values. Values 1-7 are those obtained for completed projects – the so-called historic values. Values 8-10 have been presented by experts and they constitute the preliminary estimation and image of the possible changes in the NPV values. Each of these 10 values are characterised also by the average identity coefficient, presented in column 7 „wsp\(_i\)“. It was established as the arithmetic average of the 3 coefficients \( \text{wsp}_{E1}, \text{wsp}_{E2}, \text{wsp}_{E3} \)
\[ wsp_i = \frac{wsp_{E1} + wsp_{E2} + wsp_{E2}}{3} \], \quad (12.2) \\

where:

- \( wsp_{E1} \) – identity coefficient determined by Expert 1,
- \( wsp_{E2} \) – identity coefficient determined by Expert 2,
- \( wsp_{E3} \) – identity coefficient determined by Expert 2,
- \( wsp_i \) – average identity coefficient.

Values 1 to 7 – or historic values \( k_i \) have different identity indexes (\( wsp_{Ei} \)) depending on the expert. Values \( k_i \) from 8 to 10, or expert opinions, have been applied with identity coefficient = 0.9. This is a specific weight, against which the expert opinions were applied.

Column „\( w_i \)” contains values of coefficients divided by the sum of all coefficients.

\[ w_i = \frac{wsp_i}{\sum_{i=1}^{n} wsp_i} \] \quad (12.3)

In the examined case \( n=10 \) (7 historic values + 3 values from experts)

\[ \sum_{i=1}^{n} wsp_i = 8.32 \] \quad (12.4)

On the basis of this data, for a single project examined, 4 \( k \) values were defined:

- \( k_{pes} \) – the most pessimistic of all historic values or expert opinions,
- \( k_{av} \) – average value
- \( k \) – value estimated for the project examined = 1,
- \( k_{opt} \) – optimistic value

The above values should meet the following assumptions:

\[ k_{pes} = \min(k_i) \], \quad (12.5) \\
\[ k_{opt} = \max(k_i) \], \quad (12.6)
\[ k_{av} = \sum_{i=1}^{n} w_i \times k_i , \]  
(12.7)

\[ k = 1. \]  
(12.8)

**Table 12.2. Values** \( k_{pes}, k_{av}, k, k_{opt}, p \)

*Source: own work*

<table>
<thead>
<tr>
<th>( p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_{pes} )</td>
</tr>
<tr>
<td>( k_{av} )</td>
</tr>
<tr>
<td>( k )</td>
</tr>
<tr>
<td>( k_{opt} )</td>
</tr>
</tbody>
</table>

\( p \)- level of membership of values \( k_{pes}, k_{av}, k, k_{opt} \)

On the basis of four values \( (k_{pes}, k_{av}, k, k_{opt}) \), a chart was established, which presents the fuzzy value NPV for the project planned. In the chart, the key values are \( k \) and \( k_{av} \), and therefore their membership level = 1. On the other hand, \( k_{pes} \) and \( k_{opt} \), are extreme values and they only inform us that a situation like this took place; therefore, their membership level has been determined as zero. Nevertheless, drawing a line on the chart between \( k_{pes} \) and \( k_{av} \), or \( k \) and \( k_{opt} \), we obtained information about the less significant events. This informs us that such situations as emergence of values between \( k_{pes} \) and \( k_{av} \), or \( k \) and \( k_{opt} \) should be treated as the more probable, the closer these values are to \( k_{av} \) or \( k \).

In the example presented, the investor is to select from among five projects. Therefore, characteristic points were established for each of the five projects in tabular form.
Fig. 3.1 A chart presenting NPV as a fuzzy set  
Source: own work

Table 3.3 Presentation of investment project characteristic points  
Source: own work

<table>
<thead>
<tr>
<th>i</th>
<th>k_{pes}</th>
<th>k_{av}</th>
<th>k_{opt}</th>
<th>NPV_{pes}</th>
<th>NPV_{av}</th>
<th>NPV_{def}</th>
<th>NPV_{opt}</th>
<th>NPV_{av}</th>
<th>NPV_{def}</th>
<th>NPV_{opt}</th>
<th>NPV_{av}</th>
<th>NPV_{def}</th>
<th>NPV_{opt}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.66</td>
<td>0.72</td>
<td>1.08</td>
<td>50.02</td>
<td>54.63</td>
<td>75.45</td>
<td>81.69</td>
<td>4.60</td>
<td>20.82</td>
<td>6.24</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.89</td>
<td>0.95</td>
<td>1.02</td>
<td>75.40</td>
<td>79.88</td>
<td>84.31</td>
<td>85.64</td>
<td>4.48</td>
<td>4.43</td>
<td>1.33</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.63</td>
<td>0.89</td>
<td>1.03</td>
<td>38.31</td>
<td>53.80</td>
<td>60.74</td>
<td>62.82</td>
<td>15.49</td>
<td>6.94</td>
<td>2.08</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.56</td>
<td>0.91</td>
<td>1.03</td>
<td>36.83</td>
<td>59.56</td>
<td>65.33</td>
<td>67.06</td>
<td>22.73</td>
<td>5.77</td>
<td>1.73</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.68</td>
<td>0.90</td>
<td>1.03</td>
<td>48.67</td>
<td>64.13</td>
<td>71.21</td>
<td>73.33</td>
<td>15.46</td>
<td>7.08</td>
<td>2.12</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Where:

\[ k_{pes} = \frac{NPV_{pes}}{NPV_{def}} \]  \hspace{1cm} (12.9)

\[ k_{opt} = \frac{NPV_{opt}}{NPV_{def}} \]  \hspace{1cm} (12.10)

\[ k_{av} = \frac{NPV_{av}}{NPV_{def}} \]  \hspace{1cm} (12.11)

**Fig. 12.2. A chart of characteristic values of NPV**

Source: own work

In fig. 12.2, the horizontal axis represents the project number from 1 to 5. The columns represent values NPVpes, NPVav-NPVpes, NPVdef-NPVav, NPVopt-NPVdef.
12.4 CONCLUSIONS

On the basis of comparison of 5 columns, representing investment 1,2,…,5, many conclusions can be drawn. Definitely, investment project 2 is the best – it has the highest NPV values and it differs only slightly from the average of all historic values, meaning that for a project of this type, NPV value is susceptible only to slight changes. Among all historic values, the minimum value was as much as 0.89 of the expected value, meaning that upon the extremely unfavourable conditions, the NPV value did not decrease so much in comparison with the remaining projects.

The situation is different in the case of investment project 1. Despite a relatively high NPV value, we have to take into account the possibility of substantial changes in the NPV value, since the average value established on the basis of historic values and expert opinions amounted only to 0.72 of the estimated NPV, and the NPV value in the extremely unfavourable situation was 0.66. By way of consolation, it can be said that in the case of this project, underestimation of NPV often took place, which is proven by the highest value of coefficient \( k_{opt} = 1.08 \).

Projects 3, 4 and 5 have similar \( NPV_{av} \) values in relation to the estimated values. Therefore, the best criterion for comparison of the 3 investment project will be comparison of only their estimated NPV, and in this case, project 5 is the best of the three, which also has the highest pessimistic value in relation to the estimated value, of as much as 0.68.

12.5 THE PROCEDURE ALGORITHM
– A BLOCK DIAGRAM

The procedure described is presented below in a block diagram format (Fig. 12.3.)
Fig. 12.3. A block diagram of the project selection analysis based on the effectiveness of a past project with fuzzy representation
Source: own work
CHAPTER 13

ASSESSMENT OF THE INVESTMENT PROJECT AT THE STAGE OF A FEASIBILITY STUDY WITH REGARD TO RISK AND PROBABILITY OF REACHING OF THE NPV VALUE (A.MINASOWICZ)

13.1 ABSTRACT

In order to assess an investment project on the basis of risk allocation in individual cash flows associated with individual groups of works, a procedure was proposed to determine risk in quantitative format and to establish the probability of attaining the assumed NPV value. In the method presented, expert knowledge was used, as well as calculus of probability and the fuzzy sets theory. The net present value consists of a number of discounted flows. Each of these, prior to commencement of the project, is assigned the possible deviation from the assumed value. In the procedure discussed, the probability of occurrence of a given discounted value was introduced to enable determination of risk as a function of value of the change and its probability. Using the components of the fuzzy set theory, it is possible to use expert knowledge, which, delivered verbally, can be turned into numerical values. The effect of the entire procedure is to establish a specific value of risk, associated with each flow, as well as the entire project, and a definition of the probability of occurrence of the established NPV value. On the basis of such results, it is
possible to assess the project clearly in the integrated risk and value management approach at a given stage, or to review the project risks and to establish the guidelines for their reduction.

13.2 INTRODUCTION

Every investment project, according to the NPV method, consists of specific cash flows – receipts or expenditures. These flows give us the expected NPV value. If we do not take the risk into account, individual flows can be presented as in Fig. 13.1. We do not take into account the impact of change of the discounting factor, on the assumption that the flows have already been discounted.

In figure 13.1, the horizontal axis represents index (i) of a given flow. Vectors pointing down mean expenditures, vectors pointing up mean revenues. The length of the arrow indicates the flow quantity. Exact flow values are provided in column two of table 13.1. Expenditures ($K_0$) are represented by negative values, while revenues ($Z_0$) are represented by positive values. In general, expenditures and revenues were marked as $S_0$ – cash flows. (cf tab. 13.1.)
13.3 DESCRIPTION OF THE METHOD

In the example presented, a construction project, divided into stage-related tasks, has been analysed. For each task, values of the most probable $S_0$ flows have been determined, which symbolise vectors in Fig. 13.1. In table 13.1, flows are expressed in millions of PLN.

Fig. 13.1. Individual cash flows excluding risks.
Source: own work
Table 13.1 The estimated cash flows for the project  
Source: own work

<table>
<thead>
<tr>
<th>i</th>
<th>Task</th>
<th>S₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Contract proc.</td>
<td>-1.8</td>
</tr>
<tr>
<td>2</td>
<td>Project</td>
<td>-2.1</td>
</tr>
<tr>
<td>3</td>
<td>Measurements</td>
<td>-1.3</td>
</tr>
<tr>
<td>4</td>
<td>Earthworks</td>
<td>-1.6</td>
</tr>
<tr>
<td>5</td>
<td>Boarding</td>
<td>-2.2</td>
</tr>
<tr>
<td>6</td>
<td>Concrete works</td>
<td>-3.2</td>
</tr>
<tr>
<td>7</td>
<td>Steel structures</td>
<td>-3.8</td>
</tr>
<tr>
<td>8</td>
<td>Water supply</td>
<td>-2.5</td>
</tr>
<tr>
<td>9</td>
<td>Ventilation</td>
<td>-1.8</td>
</tr>
<tr>
<td>10</td>
<td>Power supply</td>
<td>-2.8</td>
</tr>
<tr>
<td>11</td>
<td>Sewage</td>
<td>-2</td>
</tr>
<tr>
<td>12</td>
<td>Revenues year 1</td>
<td>9</td>
</tr>
<tr>
<td>13</td>
<td>Revenues year 2</td>
<td>9.5</td>
</tr>
<tr>
<td>14</td>
<td>Revenues year 3</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td><strong>NPV</strong></td>
<td><strong>3.4</strong></td>
</tr>
</tbody>
</table>

Assuming earlier that the flows of expenditures and revenues are discounted, values were summed up to obtain the NPV value. This value for the project presented amounted to PLN 3.4 million.

Although the flows calculated in the project are presented as the most probable, their probability will not amount to 1. In other words, we are not 100 per cent sure that a given flow will assume the value of $S₀$, and not greater or less. This is due to differences between the expected (most probable) values of cash flows and the actual values of these flows that result from the project implementation. These differences can be caused e.g. by:

- Improper estimation of cost of specific works;
- Non-qualified team;
- Inadequate selection of technology;
- Insufficient equipment quality;
- Other external factors not dependent on the contractor.
Therefore, it is a good idea to present the flow as a random value through its probability distribution and determine the maximum probability corresponding with flow $S_0$ and the value of cash flows of lower probability. In the assumptions presented, a triangular distribution has been applied. The peak of the triangle is value $S_0$, which has the highest probability. The vertical axis of the graph is probability, and the horizontal axis – the flow values.

Assumption of this distribution can be justified by stating that at the stage of the feasibility study, we use approximate data and application of more precise distributions at this stage is not justified. Extreme values in the triangular distribution presented are, in fact, characterised by such low probability that at the stage of feasibility study, their probability has been assumed to be equal to zero.

Fig. 13.2. Presentation of the triangular flow distribution.
Source: own work

In Fig. 13.2, variables have the following meanings:
- $Z_{pes}$ – extremely pessimistic value of revenues, assuming probability of $= 0$,
- $Z_{opt}$ – extremely optimistic value of revenues, assuming probability of $= 0$,
- $\Delta Z_{pes}$ – difference between the most probable value and the extreme pessimistic value of revenues,
\[ \Delta Z_{\text{opt}} \] – difference between the extreme optimistic value and the most probable value of revenues,

\[ K_{\text{pes}} \] – extremely pessimistic value of expenditures, assuming probability of \( = 0 \),

\[ K_{\text{opt}} \] – extremely optimistic value of expenditures, assuming probability of \( = 0 \),

\[ \Delta K_{\text{pes}} \] – difference between the most probable value and the extreme pessimistic value of expenditures,

\[ \Delta K_{\text{opt}} \] – difference between the extreme optimistic value and the most probable value of expenditures

\[ p_{0z} \] – probability of revenues assuming the value of \( Z_0 \),

\[ p_{0k} \] – probability of expenditures assuming the value of \( K_0 \).

The arms of the triangular distribution show how dramatically a revenue or expenditure may change. The character of this change is determined by tangents of the inclination angle \( \alpha \) and \( \beta \).

Angle \( \alpha \) will represent the "risk of increasing expenditures" or "risk of decreasing revenues", and angle \( \beta \) "risk of decreasing expenditures" or "risk of increasing revenues". In other words \( \alpha \) is a negative risk (associated with reduction of NPV), \( \beta \) is a positive risk (associated with increase in NPV). The risk is understood as the ability, or, rather, susceptibility to changes of individual cash flows. A high risk of change in the expenditure means a substantial change in expenditures accompanied by a slight change in probability. It means that the most probable value may be subject to substantial changes, while the external conditions change only slightly. On the other hand, a low risk of changes will be characteristic for more certain tasks less susceptible to changes. Moreover, a high risk of changes determines the lower probability of occurrence of expenditure \( K_0 \), which means we must take into account the possibility of substantial changes during implementation of the project in the case of a flow with high \( \alpha \) or \( \beta \).

In general, the risk presented as \( \alpha \) or \( \beta \) will be the function of probability and quantity of revenues or expenditures.

In theory, \( \alpha + \beta \) must be within the interval \( <0^\circ,180^\circ> \). It means that if \( \alpha + \beta \) equals to \( 0^\circ \), there is no risk associated with a given cost. It is then certain that a given flow amounts to \( S_0 \), and \( p_0 = 1 \). Such status has been described as no-risk. On the other hand, when \( \alpha + \beta \) are equal to \( 180^\circ \), it means that flow \( S_0 \) will be assumed to have the probability equal to 0.
In the further part of these calculations, as it has been mentioned before, changes in the revenues and expenditures are treated in the same way; therefore, for the sake of generalisation, the term ”revenue” or ”expenditure” has been replaced with flow $S_0$, and the probability of occurrence of expenditure $p_{0k}$ or revenue $p_{0z}$ has been replaced with $p_0$.

Using the random variable probability distribution with regard to a cash flow, the following have been determined:

$$tg(\alpha) = \frac{\Delta S_{pes}}{p_0},$$  \hspace{1cm} (13.1)

$$\alpha = arctg\left(\frac{\Delta S_{pes}}{p_0}\right),$$  \hspace{1cm} (13.2)

$$tg(\beta) = \frac{\Delta S_{opt}}{p_0},$$  \hspace{1cm} (13.3)

$$\beta = arctg\left(\frac{\Delta S_{opt}}{p_0}\right).$$  \hspace{1cm} (13.4)

Thus, to determine the risk measures – angles $\alpha$ and $\beta$, the following are needed:

To determine angle $\alpha$:
- $\Delta S_{pes}$
- $p_0$

To determine angle $\beta$:
- $\Delta S_{opt}$
- $p_0$

Value $S_0$ is received from the preliminary estimation of investment. Probability $p_0$ of the flow assuming the value of $S_0$, will be received from experts. To determine $\alpha$ or $\beta$, we need the ratio of value
\[ \frac{\Delta S_+}{\Delta S_-} = x \]  
(13.5)

\( \Delta S_+ \) and \( \Delta S_- \) are value deviations based on preliminary estimations.

Information on \( p_0 \) can be delivered by experts in linguistic format as one of the five terms:
1. Minimum = 0.75;
2. Low = 0.8;
3. Average = 0.85;
4. High = 0.9;
5. Maximum = 0.95.

Obtaining such replies, we have to translate them into numerical values. In this place, we have used a representation of linguistic space. The linguistic space, in our case, consists of fuzzy sets {minimum; low; average; high; maximum}. The numerical values of linguistic variables in case \( p_0 \) have been presented in Fig. 13.3:

![Fig. 13.3. Presentation of probability as a linguistic variable](image)

Source: own work

In the example presented, the following have been assumed as probability values of level of affinity 1 for the respective linguistic terms:
1. Minimum = 0.75;
2. Low = 0.8;
3. Average = 0.85;
4. High = 0.9;
5. Maximum = 0.95.

For a random variable, the sum of probabilities of all events must be equal to one. If the random variable has a continuous probability distribution, it means that:

\[
\int_{-\infty}^{+\infty} p(S) dS = 1
\]  

(13.6)

In the case of a continuous triangular distribution, the sum of probability of all random events associated with the flows assumed is to be recorded as follows:

\[
\int_{S_{pes}}^{S_o} \frac{p_0}{S_0 - S_{pes}} \cdot S - \frac{S_{pes} \cdot p_0}{S_0 - S_{pes}} dS + \int_{S_{opt}}^{S_0} \frac{-p_0}{S_0 - S_{opt}} \cdot S + \frac{S_{opt} \cdot p_0}{S_0 - S_{opt}} dS =
\]

\[
= \frac{1}{2} \left( \Delta S_{pes} + \Delta S_{opt} \right) \cdot p_0 = P_{\Delta}
\]

(13.7)

As we can see, it is the area of a triangle, consisting of the distribution arms and the horizontal axis.

As the distribution is triangular, we can take advantage of the fact that:

\[
\frac{\Delta S_+}{\Delta S} = \frac{\Delta S_{opt}}{\Delta S_{pes}} = x \Rightarrow \Delta S_{opt} = x \cdot \Delta S_{pes}
\]

(13.8)

Applying figure (13.7) to this, the following has been received:

\[
P_{\Delta} = \frac{1}{2} \left( \Delta S_{pes} + x \cdot \Delta S_{pes} \right) \cdot p_0 = \frac{\Delta S_{pes}}{2} \cdot (x + 1) \cdot p_0
\]

(13.9)

\[
P_{\Delta} = 1 \Rightarrow \frac{\Delta S_{pes}}{2} \cdot (1 + x) \cdot p_0 = 1
\]

(13.10)

Transformation of the formula has resulted in:

\[
\Delta S_{pes} = \frac{2}{(1 + x) \cdot p_0}
\]

(13.11)

Presented below is the linguistic format data on probability \( p_0 \), received from three experts
### Table 13.2. A breakdown of expert opinions on $p_0$ in linguistic format

Source: own work

<table>
<thead>
<tr>
<th>i</th>
<th>Task</th>
<th>$S_0$</th>
<th>$p_0$(Exp1)</th>
<th>$p_0$(Exp2)</th>
<th>$p_0$(Exp3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Contract proc.</td>
<td>-1.8</td>
<td>average</td>
<td>average</td>
<td>low</td>
</tr>
<tr>
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<td>Project</td>
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<td>average</td>
<td>low</td>
<td>minim</td>
</tr>
<tr>
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<td>Measurements</td>
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<td>high</td>
<td>high</td>
</tr>
<tr>
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<td>Earthworks</td>
<td>-1.6</td>
<td>average</td>
<td>high</td>
<td>average</td>
</tr>
<tr>
<td>5</td>
<td>Boarding</td>
<td>-2.2</td>
<td>average</td>
<td>average</td>
<td>low</td>
</tr>
<tr>
<td>6</td>
<td>Concrete works</td>
<td>-3.2</td>
<td>low</td>
<td>low</td>
<td>low</td>
</tr>
<tr>
<td>7</td>
<td>Steel structures</td>
<td>-3.8</td>
<td>average</td>
<td>average</td>
<td>low</td>
</tr>
<tr>
<td>8</td>
<td>Water supply</td>
<td>-2.5</td>
<td>average</td>
<td>high</td>
<td>high</td>
</tr>
<tr>
<td>9</td>
<td>Ventilation</td>
<td>-1.8</td>
<td>high</td>
<td>average</td>
<td>average</td>
</tr>
<tr>
<td>10</td>
<td>Power supply</td>
<td>-2.8</td>
<td>average</td>
<td>high</td>
<td>high</td>
</tr>
<tr>
<td>11</td>
<td>Sewage</td>
<td>-2</td>
<td>high</td>
<td>high</td>
<td>average</td>
</tr>
<tr>
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<td>Revenues year 1</td>
<td>9</td>
<td>average</td>
<td>high</td>
<td>average</td>
</tr>
<tr>
<td>13</td>
<td>Revenues year 2</td>
<td>9.5</td>
<td>average</td>
<td>average</td>
<td>low</td>
</tr>
<tr>
<td>14</td>
<td>Revenues year 3</td>
<td>10</td>
<td>average</td>
<td>low</td>
<td>average</td>
</tr>
</tbody>
</table>

On the basis of the linguistic space established, numerical values of probability $p_0$ determined by experts were established.
Table 13.3. A breakdown of expert opinions on $p_0$ in numerical format for points with membership level 1
Source: own work

<table>
<thead>
<tr>
<th>i</th>
<th>Task</th>
<th>$S_0$</th>
<th>$p_0$(Exp1)</th>
<th>$p_0$(Exp2)</th>
<th>$p_0$(Exp3)</th>
<th>$p_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Contract proc.</td>
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<td>0.85</td>
<td>0.8</td>
<td>0.83</td>
</tr>
<tr>
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<td>Project</td>
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<td>0.8</td>
<td>0.75</td>
<td>0.80</td>
</tr>
<tr>
<td>3</td>
<td>Measurements</td>
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<td>0.9</td>
<td>0.92</td>
</tr>
<tr>
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<td>Earthworks</td>
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<td>0.9</td>
<td>0.85</td>
<td>0.87</td>
</tr>
<tr>
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<td>0.85</td>
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<td>0.83</td>
</tr>
<tr>
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<td>Concrete works</td>
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<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.80</td>
</tr>
<tr>
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<td>Steel structures</td>
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<td>0.85</td>
<td>0.85</td>
<td>0.8</td>
<td>0.83</td>
</tr>
<tr>
<td>8</td>
<td>Water supply</td>
<td>-2.5</td>
<td>0.85</td>
<td>0.9</td>
<td>0.9</td>
<td>0.88</td>
</tr>
<tr>
<td>9</td>
<td>Ventilation</td>
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<td>0.9</td>
<td>0.85</td>
<td>0.85</td>
<td>0.87</td>
</tr>
<tr>
<td>10</td>
<td>Power supply</td>
<td>-2.8</td>
<td>0.85</td>
<td>0.9</td>
<td>0.9</td>
<td>0.88</td>
</tr>
<tr>
<td>11</td>
<td>Sewage</td>
<td>-2</td>
<td>0.9</td>
<td>0.9</td>
<td>0.85</td>
<td>0.88</td>
</tr>
<tr>
<td>12</td>
<td>Revenues year 1</td>
<td>9</td>
<td>0.85</td>
<td>0.9</td>
<td>0.85</td>
<td>0.87</td>
</tr>
<tr>
<td>13</td>
<td>Revenues year 2</td>
<td>9.5</td>
<td>0.85</td>
<td>0.85</td>
<td>0.8</td>
<td>0.83</td>
</tr>
<tr>
<td>14</td>
<td>Revenues year 3</td>
<td>10</td>
<td>0.85</td>
<td>0.8</td>
<td>0.85</td>
<td>0.83</td>
</tr>
</tbody>
</table>

Value $p_0$ is the arithmetic average of information on $p_0$ from 3 Experts: $[p_0$(Exp1), $p_0$(Exp2), $p_0$(Exp3)]
Another set of information, which can be obtained on the basis of historic data, is the following ratio:

$$\frac{\Delta S_+}{\Delta S_-} = x \quad (13.12)$$
Table 13.4. A breakdown of the value of ratio $\frac{\Delta S^+_i}{\Delta S^-_i} = x$

Source: own work

<table>
<thead>
<tr>
<th>Task</th>
<th>$S_0$</th>
<th>$x$</th>
<th>$p_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Expenditures</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 Contract proc.</td>
<td>-1.8</td>
<td>0.4</td>
<td>0.83</td>
</tr>
<tr>
<td>2 Project</td>
<td>-2.1</td>
<td>0.6</td>
<td>0.80</td>
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<tr>
<td>3 Measurements</td>
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<td>0.5</td>
<td>0.92</td>
</tr>
<tr>
<td>4 Earthworks</td>
<td>-1.6</td>
<td>0.5</td>
<td>0.87</td>
</tr>
<tr>
<td>5 Boarding</td>
<td>-2.2</td>
<td>0.8</td>
<td>0.83</td>
</tr>
<tr>
<td>6 Concrete works</td>
<td>-3.2</td>
<td>0.5</td>
<td>0.80</td>
</tr>
<tr>
<td>7 Steel structures</td>
<td>-3.8</td>
<td>0.7</td>
<td>0.83</td>
</tr>
<tr>
<td>8 Water supply</td>
<td>-2.5</td>
<td>0.8</td>
<td>0.88</td>
</tr>
<tr>
<td>9 Ventilation</td>
<td>-1.8</td>
<td>1</td>
<td>0.87</td>
</tr>
<tr>
<td>10 Power supply</td>
<td>-2.8</td>
<td>0.7</td>
<td>0.88</td>
</tr>
<tr>
<td>11 Sewage</td>
<td>-2</td>
<td>0.6</td>
<td>0.88</td>
</tr>
<tr>
<td><strong>Revenues</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>12 Revenues year 1</td>
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<td>0.7</td>
<td>0.87</td>
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<td>13 Revenues year 2</td>
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<td>0.83</td>
</tr>
<tr>
<td>14 Revenues year 3</td>
<td>10</td>
<td>0.9</td>
<td>0.83</td>
</tr>
</tbody>
</table>

Characteristic values of each of flow distributions and their relative values in relation to absolute value from $S_0$ have also been presented for the sake of better representation of deviations of cash flows from the most probable values. On the basis of relative values, relative distributions of cash flows were also prepared.
Table 13.5. Typical values for each distribution
Source: own work

<table>
<thead>
<tr>
<th>Task</th>
<th>Contract proc.</th>
<th>Project</th>
<th>Measurements</th>
<th>Earthworks</th>
<th>Concrete works</th>
<th>Steel structures</th>
<th>Water supply</th>
<th>Ventilation</th>
<th>Power supply</th>
<th>Sewage</th>
<th>Revenues year 1</th>
<th>Revenues year 2</th>
<th>Revenues year 3</th>
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<td>0.92</td>
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Expenditures

<table>
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</thead>
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<td>14.0</td>
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<td>14.0</td>
<td>14.0</td>
<td>14.0</td>
<td>14.0</td>
</tr>
<tr>
<td>Revenues year 2</td>
<td>13.0</td>
<td>13.0</td>
<td>13.0</td>
<td>13.0</td>
<td>13.0</td>
<td>13.0</td>
<td>13.0</td>
<td>13.0</td>
<td>13.0</td>
<td>13.0</td>
<td>13.0</td>
<td>13.0</td>
<td>13.0</td>
</tr>
<tr>
<td>Revenues year 3</td>
<td>12.0</td>
<td>12.0</td>
<td>12.0</td>
<td>12.0</td>
<td>12.0</td>
<td>12.0</td>
<td>12.0</td>
<td>12.0</td>
<td>12.0</td>
<td>12.0</td>
<td>12.0</td>
<td>12.0</td>
<td>12.0</td>
</tr>
</tbody>
</table>
Values in individual columns have been calculated on the basis of formulas:

\[ \Delta S_{pes} = \frac{2}{(1 + x) \cdot p_0} \]  
(13.13)

\[ \Delta S_{opt} = x \cdot \Delta S_{pes} \]  
(13.14)

\[ S_{pes} = S_0 - \Delta S_{pes} \]  
(13.15)

\[ S_{opt} = S_0 + \Delta S_{opt} \]  
(13.16)

In the last four columns of table 13.5, relative values were established

\[ \frac{\Delta S_{pes}}{|S_0|}, \frac{\Delta S_{opt}}{|S_0|}, \frac{S_{pes}}{|S_0|}, \frac{S_{opt}}{|S_0|} \]. This allowed for determination of \( \alpha \) and \( \alpha_w \) and \( \beta \) and \( \beta_w \). These values were calculated on the basis of the following formulas and presented in table 13.5.

\[ \tan(\alpha) = \frac{\Delta S_{pes}}{p_0} \]  
(13.17)

\[ \alpha = \arctan\left(\frac{\Delta S_{pes}}{p_0}\right) \]  
(13.18)

\[ \tan(\alpha_w) = \frac{\frac{\Delta S_{pes}}{|S_0|}}{p_0} = \frac{\Delta S_{pes}}{|S_0| \cdot p_0} \]  
(13.19)

\[ \alpha_w = \arctan\left(\frac{\Delta S_{pes}}{|S_0| \cdot p_0}\right) = \arctan\left(\frac{\Delta S_{pes}}{|S_0| \cdot p_0}\right) \]  
(13.20)
\[ \tan(\beta) = \frac{\Delta S_{\text{opt}}}{p_0} \]  
(13.21)

\[ \beta = \arctan\left(\frac{\Delta S_{\text{opt}}}{p_0}\right) \]  
(13.22)

\[ \Delta = \begin{pmatrix} p \end{pmatrix} \begin{pmatrix} \Delta S_{\text{opt}} \\ \beta \end{pmatrix} \]  
(13.23)

\[ \beta_w = \arctan\left(\frac{\Delta S_{\text{opt}}}{\sqrt{S_0} \cdot p_0}\right) \]  
(13.24)

Values \( \alpha \) and \( \beta \) are characteristic for our initial distributions. On the other hand, values \( \alpha_w \) and \( \beta_w \) are characteristic for relative distributions, in which on the horizontal axis we get \( \frac{S_i}{S_0} \) – the ratio of flow to its value established in the project. In this way, values \( \alpha_w \) and \( \beta_w \) characterise the relative value.
In this way, each cash flow has been presented as two distributions – absolute and relative. Fig. 13.5 presents an exemplary absolute and relative distribution for one of the stages, that is, 1. Contract proc.

Fig. 13.5. Presentation of an exemplary flow in form of a triangular probability distribution in separate systems of coordinates
Source: own work
Table 13.6. A breakdown of values $\alpha$, $\beta$, $\alpha_w$, $\beta_w$
Source: own work

<table>
<thead>
<tr>
<th>i</th>
<th>Task</th>
<th>$\text{tg}(\alpha)$</th>
<th>$\text{tg}(\beta)$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\text{tg}(\alpha_w)$</th>
<th>$\text{tg}(\beta_w)$</th>
<th>$\alpha_w$</th>
<th>$\beta_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Contract proc.</td>
<td>2.06</td>
<td>0.82</td>
<td>64</td>
<td>39</td>
<td>1.14</td>
<td>0.46</td>
<td>49</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>Project</td>
<td>1.95</td>
<td>1.17</td>
<td>63</td>
<td>50</td>
<td>0.93</td>
<td>0.56</td>
<td>43</td>
<td>29</td>
</tr>
<tr>
<td>3</td>
<td>Measurements</td>
<td>1.59</td>
<td>0.79</td>
<td>58</td>
<td>38</td>
<td>1.22</td>
<td>0.61</td>
<td>51</td>
<td>31</td>
</tr>
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<td>1.78</td>
<td>0.89</td>
<td>61</td>
<td>42</td>
<td>1.11</td>
<td>0.55</td>
<td>48</td>
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<tr>
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<td>Boarding</td>
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<td>1.28</td>
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<td>0.58</td>
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<td>1.04</td>
<td>64</td>
<td>46</td>
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<td>0.45</td>
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<td>8</td>
<td>Water supply</td>
<td>1.42</td>
<td>1.14</td>
<td>55</td>
<td>49</td>
<td>0.57</td>
<td>0.46</td>
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<td>25</td>
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<tr>
<td>9</td>
<td>Ventilation</td>
<td>1.33</td>
<td>1.33</td>
<td>53</td>
<td>53</td>
<td>0.74</td>
<td>0.74</td>
<td>37</td>
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<td>Power supply</td>
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<td>1.06</td>
<td>56</td>
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<td>0.54</td>
<td>0.38</td>
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<td>21</td>
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<td>11</td>
<td>Sewage</td>
<td>1.60</td>
<td>0.96</td>
<td>58</td>
<td>44</td>
<td>0.80</td>
<td>0.48</td>
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<td>26</td>
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<td>Revenues year 1</td>
<td>1.57</td>
<td>1.10</td>
<td>57</td>
<td>48</td>
<td>0.17</td>
<td>0.12</td>
<td>10</td>
<td>7</td>
</tr>
<tr>
<td>13</td>
<td>Revenues year 2</td>
<td>1.60</td>
<td>1.28</td>
<td>58</td>
<td>52</td>
<td>0.17</td>
<td>0.13</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>14</td>
<td>Revenues year 3</td>
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<td>57</td>
<td>54</td>
<td>0.15</td>
<td>0.14</td>
<td>9</td>
<td>8</td>
</tr>
</tbody>
</table>

Fig. 13.6. Presentation – in column format – of value $\alpha$, $\beta$
Source: own work
Fig. 13.7. Presentation – in column format – of value $\alpha_w$, $\beta_w$
Source: own work

On the basis of cash flows presented as random variables, NPV has been established:

$$NPV_{pes} = \sum_{i=1}^{n} S_{pes}$$  \hspace{1cm} (13.25)

$$NPV_0 = \sum_{i=1}^{n} S_0$$  \hspace{1cm} (13.26)

$$NPV_{opt} = \sum_{i=1}^{n} S_{opt}$$  \hspace{1cm} (13.27)

Values provided in table 13.7 calculated on the basis of formulas (13.25) - (13.27) are provided in millions of PLN. Due to the fact that the NPV value has been presented as a random variable, the value $p_0$ of its occurrence and value of $\alpha$, $\beta$, $\alpha_w$, $\beta_w$ can be determined.
\[ P_\Delta = \frac{1}{2} \cdot (NPV_{opt} - NPV_{pes}) \cdot p_0 = 1 \Rightarrow p_0 = \frac{2}{NPV_{opt} - NPV_{pes}} \quad (13.28) \]

Table 13.7. Presentation of characteristic values of NPV
Source: own work

<table>
<thead>
<tr>
<th>NPV$_{pes}$</th>
<th>-16.39</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV$_0$</td>
<td>3.40</td>
</tr>
<tr>
<td>NPV$_{opt}$</td>
<td>16.50</td>
</tr>
<tr>
<td>$p_0$</td>
<td>0.061</td>
</tr>
<tr>
<td>$\tan(\alpha)$</td>
<td>325.6</td>
</tr>
<tr>
<td>$\tan(\beta)$</td>
<td>215.5</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>89.824</td>
</tr>
<tr>
<td>$\beta$</td>
<td>89.734</td>
</tr>
</tbody>
</table>

Fig. 13.8. Presentation of NPV value as a random variable system
Source: own work
Analogically to creation of absolute values of individual flows, a relative distribution for NPV value has been established.

**Table 13.8. Presentation of NPV values**

Source: own work

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV_{pes}/NPV_0</td>
<td>-4.82</td>
</tr>
<tr>
<td>NPV_0/NPV_0</td>
<td>1</td>
</tr>
<tr>
<td>NPV_{opt}/NPV_0</td>
<td>4.85</td>
</tr>
<tr>
<td>p_0</td>
<td>0.061</td>
</tr>
<tr>
<td>tg((\alpha_w))</td>
<td>95.76</td>
</tr>
<tr>
<td>tg((\beta_w))</td>
<td>63.39</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>89.402</td>
</tr>
<tr>
<td>(\beta)</td>
<td>89.096</td>
</tr>
</tbody>
</table>

**Fig. 13.9. Relative NPV value**

Source: own work
As can be seen, the NPV value has been significantly extended. This is due to the distribution assumed and the mode of determination of NPV; nevertheless, it provides information on NPV and the risk associated with obtaining of a lower or higher value. Thanks to this mode of presentation, it is easy to compare the investment projects and the risks associated with each of them.

The values of all distributions were established not on the basis of the entire set of assessment of linguistic values \( p_0 \), but only on the values of membership level equal to 1, that is:

- minimum = 0.75;
- low = 0.80;
- average = 0.85;
- high = 0.9;
- maximum = 0.95.

Thus, the distribution values represent acute distributions. To take into account the general subjective nature and lack of precision of expression of \( p_0 \), it is necessary to examine all values of the linguistic variable. Thanks to assuming of the triangular membership function, it is sufficient to conduct calculations for three values:

- \( L \) – extreme left of membership level = 0;
- \( C \) – central of membership level = 1;
- \( P \) – extreme right of membership level = 0.

In our case, linguistic value (L ; C ; P) will be as follows:

- minimum = (0.7 ; 0.75 ; 0.8);
- low = (0.75 ; 0.80 ; 0.85);
- average = (0.85 ; 0.90 ; 0.95);
- high = (0.85 ; 0.9 ; 0.95);
- maximum = (0.9 ; 0.95 ; 1)

In this way, instead of one probability value \( p_0 \), we received three values \( p_0 \), and thus three distribution values for a single cash flow (extreme left – L, central – C, extreme right – P).
On the basis of such flow distributions, the fuzzy NPV distribution was established.
Fig. 13.12. Absolute NPV distribution
Source: own work

Fig. 13.13. Relative NPV distribution
Source: own work

Summing up, in the example presented, probability and statistics calculations were used to illustrate the cash flows. The author’s objective was to present not only the value of flows, but also the probability of their occurrence. This allowed for characterisation of the risk. For the sake of clarity, a triangular probability distribution was applied.
13.4 CONCLUSIONS

There are many theories on risk. One of the most accurate of these, according to the author, states that risk is a function of two variables: the probability of failure (success) and the quantity of loss (profit). Thanks to presentation of cash flows as triangular distributions, risk was presented as angles $\alpha$ and $\beta$

$$\alpha = \arctg\left(\frac{\Delta S_{pes}}{p_0}\right) - \text{risk of success}$$

$$\beta = \arctg\left(\frac{\Delta S_{opt}}{p_0}\right) - \text{risk of failure}$$

These angles are the functions of revenues (expenditures) and the probability of occurrence of the expected flow in project $S_0$. The higher is the value of change of this stream, the greater is $\alpha$ or $\beta$. On the other hand, the greater the probability $p_0$, the lower the risk. It is logical, because the greater the probability of occurrence of a given flow in the project, or the more certain it is, the less risk is associated with it. Moreover, presentation in triangular format of the random variable distribution chart shows very well the level of this risk, as well as the risk of success in relation to the risk of failure.

In the example presented, some information was obtained from experts in verbal format. The best tool to translate this information to numerical format is the use of fuzzy sets. Thanks to this, the distribution of probability of the random value is presented using the area designated by two extreme distributions of membership level 1. In between these, there is a central distribution of the level of membership equal to 1.
13.5 THE PROCEDURE ALGORITHM – A BLOCK DIAGRAM

The procedure presented has been depicted in a diagram format below (Fig. 13.14)

![Block Diagram](image_url)

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**Fig. 13.14.** A block diagram of assessment of the investment project at the stage of the feasibility study with regard to risk and probability of attaining the NPV value

**Source:** own work
CHAPTER 14

ANALYSIS OF A MULTI-CRITERIA SELECTION OF THE BEST OFFER IN TENDER PROCEDURES USING EXPERT KNOWLEDGE BASED ON FUZZY PREMISES (A.MINASOWICZ, B.KOSTRZEWIA)

14.1 ABSTRACT

The presented analysis allows for selection of the project contractor, so that the potential risk, associated with a failure to keep the deadline, or the insufficient quality of works performed, is reduced to a minimum. For this purpose, it is necessary to define and specify the basic requirements to be classified as the selection criteria. Using the specific formulas, we establish standardised numerical values for all parameters of the investment project and the contractor’s offer. The next step is to establish the expert base of rules to serve as a basis for selection of the best offer. It is necessary to apply elements of the fuzzy set theory to use the expert knowledge delivered in linguistic format. Using the apparatus of mathematics, based on arithmetic of fuzzy numbers, it is possible to determine the so-called attractiveness of the offer, which serves as a basis for specification of the best offer. The method proposed can be applied at the stage of integrated value and risk management to specify the tender strategy for the project planned.
14.2 INTRODUCTION

The functioning of the tender system in many European countries, particularly in Poland, leads to delays in preparation and implementation of construction projects. One of the weaknesses of this system is that the contractor company selected is usually the one which offers the lowest price. Selection of such bidders leads to many problems during implementation of the project, including low quality of performance of the construction facility or attempts to exert pressure on the investor to obtain additional financial means for so-called “additional works”.

This article is an attempt to show the method of selection of the contractor company on the basis of a specific number of criteria, which characterise both the future contractor and the planned construction project. This knowledge is usually known to experts, and through their opinions and experience, it is possible to create tools to enable reasonable selection of whichever bidder proposes the most proper implementation of the project in accordance with the parameters defined.

In order to extract expert knowledge, linguistic assessment was applied, using fuzzy sets. The fuzzy set theory is applicable to the assessment of issues associated with imprecise and unclear information.

Proposals contained in this article may be used in the future to construct an expert system to be used for selection of the bidder, who meets the conditions established and assumptions of the construction project to the fullest extent. Application of such system would shorten the tender procedure, ensuring the appropriate quality of the bidder selection decision.
14.3 DESCRIPTION OF THE METHOD

For the purpose of objective selection of the future contractor, in the first place, the selection criteria were assessed. Two of these were considered to be sufficient: assessment of contractor reliability – \( O \), and the proposed bid price – \( C \).

Depending on the Terms of Reference, it is also possible to add other criteria, such as the warranty period. In the article presented, for the sake of clear presentation of the issue, it was assumed that this criterion would be defined in advance by the investor and it would not be subject to changes. Secondly, parameters describing the subject of the tender were defined. This parameter was referred to as the investment significance \( Z \), consisting of three components, the so-called sub-parameters; level of difficulty – \( P \); time limits – \( T \); and, the value of investment expenditures – \( K \).

On the basis of two criteria, or the assessment of reliability of the contractor and the proposed bid price, and upon the sub-parameters defined to specify the subject of the tender, it was assessed clearly, which of the contractor bids was the best for a project with a specified level of expenditures, difficulty and time limits. The mechanism used to select the best offer is the expert system proposal, in which the base of rules is constructed in linguistic format, on the basis of premises in form of fuzzy sets. In the fuzzy set theory, a component may be assigned to a set partially, meaning that its level of membership may be between 0 and 1 inclusively. The base of rules was established on the basis of a simple rule:

\[
\text{If the project significance is } \ldots \text{, the contractor’s reliability is } \ldots \text{ and the bid price is } \ldots \text{, then advantage (attractiveness) of the bid is } \ldots.\]

As an example, it was proposed to allow the experts to formulate the base of rules using only specific terms. The following terms were used in the description: marginal, average, high, key. Assessment of reliability of the contractor and the bid price is described in terms of low, average, high. Attractiveness of the offer, established by the Expert, is a numerical value between 0 to 1 to one tenth. Table 1 presents one of the possible bases of rules, established by the expert.
### Table 14.1. A breakdown of an exemplary base of rules of the expert system.
Source: own work

<table>
<thead>
<tr>
<th>No.</th>
<th>Project significance</th>
<th>Contractor score</th>
<th>Bid price</th>
<th>Bid attractiveness</th>
</tr>
</thead>
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<td>1</td>
<td>marginal</td>
<td>low</td>
<td>low</td>
<td>0.7</td>
</tr>
<tr>
<td>2</td>
<td>marginal</td>
<td>low</td>
<td>average</td>
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</tr>
<tr>
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<td>marginal</td>
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<td>high</td>
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</tr>
<tr>
<td>4</td>
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<td>average</td>
<td>low</td>
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<td>average</td>
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</tr>
<tr>
<td>35</td>
<td>key</td>
<td>high</td>
<td>average</td>
<td>0.8</td>
</tr>
<tr>
<td>36</td>
<td>key</td>
<td>high</td>
<td>high</td>
<td>0.7</td>
</tr>
</tbody>
</table>
Each of these terms, such as the "marginal project significance" or "high score of the contractor" in the fuzzy sets theory is a linguistic variable, defined through its linguistic value. A set of all linguistic values, specifying a given variable, is known as the linguistic space. In the article presented, simple linguistic spaces have been applied, in which linguistic variables are described for specific intervals with linear functions. Presented below are linguistic spaces applicable to calculations, describing the following variables: project significance, bid price, contractor reliability score.

![Graphic representation of linguistic space of the term „investment project significance”](source: own work)
Fig. 14.2. Graphic representation of linguistic space of the term „contractor reliability score”
Source: own work

Fig. 14.3. Graphic representation of linguistic space of the term „bid price”
Source: own work
Linguistic space is presented using a two-dimensional Cartesian system, in which the y-axis constitutes the level of membership of a value to the set, and the x-axis consists of numerical values of linguistic variables of the investment project significance, the bid price and the contractor score. Numerical values are between 0 and 1, thanks to which the methodology of examination of contractor bids is universal. These values are relative standardised values, obtained on the basis of absolute values.

The standardised bid price value was established on the basis of absolute values according to formula

$$C_i^n = \frac{C_i - C_{\min}}{C_{\max} - C_{\min}}$$

(14.1)

where:

- $C_i^n$ – the standardised value of the bid price of the i-th contractor,
- $C_i$ – the absolute value of the bid price of the i-th subcontractor,
- $C_{\max}$ – the maximum value from the set of bids of all contractors, who participated in the tender,
- $C_{\min}$ – the minimum value from the set of bid prices of contractors, who participated in the tender.

The mode of assessment of contractor reliability should be established individually for each tender. Depending on the specific nature of the tender, the investor may specify the basic requirements, for which they will assign a specific score. Summing up the score obtained, we received the absolute value of assessment $O_i$ and, analogically to formula (1), we calculate the standardised value $O_i^n$.

The numerical value of the project significance, as it has been mentioned, has been established on the basis of three components (sub-parameters): the level of difficulty of the investment project, time limitations and the value of investment expenditures. The level of difficulty of investment $P_i^n$ has been established within the interval of 0 to 1 on the basis of expert opinions, so that 0 means the minimum level and 1 – the maximum level. Accuracy at the level of one tenth was defined. Analogously, on the basis of expert opinions, time limits $T_i^n$ were defined. Like in the case of the difficulty level, this is a value from the
interval from value 0, meaning practically no time limits, to 1 – meaning very high time limits. The last sub-parameter, or the value of investment expenditures \( K_i^n \) is specified by the investor. On the basis of his or her investment portfolio, he/she specifies the cost associated with a given investment. 0 means the minimum level, 1 – the maximum level for all tasks of the investor.

For example: \( K_i^n =0,7 \), \( T_i^n=0,6 \), \( P_i^n=0,4 \). On the basis of three standardised sub-parameters, a standardized value of investment \( Z_i^n \) was established. On the basis of this formula, the value of significance of the investment was established:

\[
Z_i^n = \frac{3K_i^n + T_i^n + 2P_i^n}{6} = \frac{3 \cdot 0,7 + 0,6 + 2 \cdot 0,4}{6} = 0,58
\]

It was assumed that six bidders participated in the bidding procedure. The prices of individual bids were as follows: \( C_1 =125 \text{ million PLN} \), \( C_2 =118 \text{ million PLN} \), \( C_3 =111 \text{ million PLN} \), \( C_4 =132 \text{ million PLN} \), \( C_5 =120 \text{ million PLN} \), \( C_6 =114 \text{ million PLN} \). On the basis of formula (1) standardized values were established: \( C_1^n =0,67 \), \( C_2^n =0,33 \), \( C_3^n =0,00 \), \( C_4^n =1,00 \), \( C_5^n =0,43 \), \( C_6^n =0,14 \).

Exemplary contractor reliability scores were assigned the following values: \( O_1 =18 \text{ pts} \), \( O_2 =14 \text{ pts} \), \( O_3 =11 \text{ pts} \), \( O_4 =16 \text{ pts} \), \( O_5 =20 \text{ pts} \), \( O_6 =17 \text{ pts} \). Analogically, on the basis of formula 1, (1) standardized values were determined: \( O_1^n =0,78 \), \( O_2^n =0,33 \), \( O_3^n =0,00 \), \( O_4^n =0,56 \), \( O_5^n =1,00 \), \( O_6^n =0,67 \).

Thus, for each bid, standardised numerical values were obtained for the bid price and contractor score criteria. For the first bid, the standardised price amounted to \( C_1^n =0,67 \), and the contractor score was \( O_1^n =0,78 \), which, translated to linguistic space of individual linguistic variables, provided specific results.
Fig. 14.4. Graphic representation of the levels of membership of numerical values to individual fuzzy sets for the proposed bid price
Source: own work

Fig. 14.5. Graphic representation of the levels of membership of numerical values to individual fuzzy sets for the contractor reliability score
Source: own work
The bid price $C_1^n=0,67$ means that at the same time this is the „average price” with the membership level equal to 0,66 and it is the „high price” with the membership level of 0,34. Analogically, the contractor score $O_1^n=0,78$ means that at the same time this is the „average score” with membership level 0,44 and „high score” with membership level 0,56. Membership levels were established analogically for individual fuzzy sets in the case of the remaining offers. These have been presented in tables 2 and 3.

Table 14.2. A breakdown of membership levels of individual numerical values to the appropriate fuzzy sets in the linguistic space for the bid price
Source: own work

<table>
<thead>
<tr>
<th>Bid price</th>
<th>value</th>
<th>Membership level 1</th>
<th>set 1</th>
<th>Membership level 2</th>
<th>Set 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1^n$</td>
<td>0,67</td>
<td>0,34</td>
<td>high</td>
<td>0,66</td>
<td>average</td>
</tr>
<tr>
<td>$C_2^n$</td>
<td>0,33</td>
<td>0,67</td>
<td>low</td>
<td>0,33</td>
<td>average</td>
</tr>
<tr>
<td>$C_3^n$</td>
<td>0,00</td>
<td>0,00</td>
<td>low</td>
<td>1,00</td>
<td>average</td>
</tr>
<tr>
<td>$C_4^n$</td>
<td>1,00</td>
<td>1,00</td>
<td>high</td>
<td>0,00</td>
<td>average</td>
</tr>
<tr>
<td>$C_5^n$</td>
<td>0,43</td>
<td>0,86</td>
<td>low</td>
<td>0,14</td>
<td>average</td>
</tr>
<tr>
<td>$C_6^n$</td>
<td>0,14</td>
<td>0,29</td>
<td>low</td>
<td>0,71</td>
<td>average</td>
</tr>
</tbody>
</table>

Table 14.3. A breakdown of membership levels of individual numerical values to the appropriate fuzzy sets in the linguistic space for the contractor score
Source: own work

<table>
<thead>
<tr>
<th>Contractor reliability score</th>
<th>value</th>
<th>Membership level 1</th>
<th>set 1</th>
<th>Membership level 2</th>
<th>Set 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_1^n$</td>
<td>0,78</td>
<td>0,56</td>
<td>high</td>
<td>0,44</td>
<td>average</td>
</tr>
<tr>
<td>$O_2^n$</td>
<td>0,33</td>
<td>0,67</td>
<td>low</td>
<td>0,33</td>
<td>average</td>
</tr>
<tr>
<td>$O_3^n$</td>
<td>0,00</td>
<td>0,00</td>
<td>low</td>
<td>1,00</td>
<td>average</td>
</tr>
<tr>
<td>$O_4^n$</td>
<td>0,56</td>
<td>0,11</td>
<td>high</td>
<td>0,89</td>
<td>average</td>
</tr>
<tr>
<td>$O_5^n$</td>
<td>1,00</td>
<td>1,00</td>
<td>high</td>
<td>0,00</td>
<td>average</td>
</tr>
<tr>
<td>$O_6^n$</td>
<td>0,67</td>
<td>0,33</td>
<td>high</td>
<td>0,67</td>
<td>average</td>
</tr>
</tbody>
</table>

The numerical value of the investment significance amounted to 0,58 meaning that its significance is „high” with membership level 0,74 and „average” with membership level 0,26.

For each bid, it is necessary to examine eight rules from the base of rules assumed, since there are three premises per each rule, where each is represented
by two fuzzy sets with the appropriate membership levels. For instance, for bid 1, it is necessary to apply the following rules:

**Table 14.4. A breakdown of rules establish to assess the attractiveness of bid 1**
Source: own work

<table>
<thead>
<tr>
<th>No.</th>
<th>Investment significance</th>
<th>Contractor score</th>
<th>Bid price</th>
<th>Bid attractiveness</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>average</td>
<td>average</td>
<td>average</td>
<td>0,6</td>
<td>0,457</td>
</tr>
<tr>
<td>15</td>
<td>average</td>
<td>average</td>
<td>high</td>
<td>0,5</td>
<td>0,346</td>
</tr>
<tr>
<td>17</td>
<td>average</td>
<td>high</td>
<td>average</td>
<td>0,8</td>
<td>0,494</td>
</tr>
<tr>
<td>18</td>
<td>average</td>
<td>high</td>
<td>high</td>
<td>0,6</td>
<td>0,383</td>
</tr>
<tr>
<td>23</td>
<td>high</td>
<td>average</td>
<td>average</td>
<td>0,8</td>
<td>0,617</td>
</tr>
<tr>
<td>24</td>
<td>high</td>
<td>average</td>
<td>high</td>
<td>0,6</td>
<td>0,506</td>
</tr>
<tr>
<td>26</td>
<td>high</td>
<td>high</td>
<td>average</td>
<td>0,8</td>
<td>0,407</td>
</tr>
<tr>
<td>27</td>
<td>high</td>
<td>high</td>
<td>high</td>
<td>0,6</td>
<td>0,296</td>
</tr>
</tbody>
</table>

Attractiveness of the bid was determined as a weighted average according to the above rules. The weights have been established on the basis of arithmetic averages of levels of affinity of individual numerical values of the contractor score, the investment significance and the bid price, to the applicable linguistic values. In this way, the bid attractiveness result for bid 1 was obtained, equal to 0,680. Attractiveness of the remaining bids was calculated in the same manner. The following results were obtained: $A_{O1} = 0,680$, $A_{O2} = 0,490$, $A_{O3} = 0,557$, $A_{O4} = 0,660$, $A_{O5} = 0,796$, $A_{O6} = 0,740$.

On the basis of results of attractiveness, the best bid was selected. In the case of specific tender parameters – the time frames, level of complexity and the value of investment expenditures, bid 5 turned out to be the most attractive. As it can be seen on the basis of the example presented, the bid winner was not the cheapest company, but a company that presented a relatively competitive bid, at the same time, obtaining the highest reliability score from the investor. For these parameters of the tender, this bid turned out to be the best.
14.4 CONCLUSIONS

The method of selection of the best bid, proposed in this article, allows for selection of the winner on the basis of more than one criterion. This method can be modified and adjusted to a specific tender e.g. by adding more criteria or their modification. Using the method proposed, the investor is able to specify clearly the order conditions, at the same time specifying the priorities which are decisive for the bid selection, on the basis of knowledge and experience of independent experts.

The further direction of development of the method proposed, in the authors’ opinion, should be creation of a standardised expert system – its base of knowledge could be used for a specific tender and for application of more advanced mathematical tools in the field of fuzzy sets theory, at the same time, optimising the result of this method.
14.5 THE PROCEDURE ALGORITHM – A BLOCK DIAGRAM

Presented below are the general rules of the procedure in a block diagram format (Fig. 14.6.).

Fig. 14.6. A block diagram of analysis of multi-criteria selection of the best bid in tender procedures using an expert system based on fuzzy premises

Source: own work
CHAPTER 15

TIME-COST ANALYSIS OF A PLANNED CONSTRUCTION PROJECT TAKING INTO ACCOUNT RISK BASED ON EXPERT KNOWLEDGE USING FUZZY SETS (A.MINASOWICZ, B.KOSTRZEWKA)

15.1 ABSTRACT

This analysis aims to provide a detailed risk review for a given project at the stage of value engineering of the integrated value and risk management VRM-3. On the basis of the cost estimate and the time schedule established, the cost or time deviations for each task are specified for individual groups of works. Expert knowledge is used for this purpose. In order to transform the input information, it is necessary to introduce fuzzy modelling, which includes fuzzification, inference and defuzzification processes. The proposed procedure allows for automatic determination of optimistic and pessimistic project scenarios with regard to both time and cost, using simple mathematical operators, like the arithmetic average and the centre of mass. In this way, we obtain the quantified risks associated with time and cost of the project, which allows for comparison of several technologies for implementation of the same project and selection of the optimum variant.
15.2 INTRODUCTION

A key stage of every construction project is its implementation, or the moment of creation of the facility. This is associated with selection of the contractor, technology of performance of works and the implementation cycle. All of these aspects influence to a specific extent the two key project parameters, which are the cost and deadline of implementation.

In this article, we present the method of introducing the risk associated with a given investment at the implementation stage, using the experience and knowledge of independent experts. We used the fuzzy set theory as a tool.

This approach towards investment project implementation allows the investor, or the general contractor, to obtain knowledge about potential changes in the time and cost of implementation.

15.3 DESCRIPTION OF THE METHOD

The example used involved the following project. The facility concerned is an apartment building and the estimated construction cost is PLN 28,346,000. The entire project consists of seventeen groups of works, such as preparatory works, earthworks etc. Specific costs, determined by the author of the cost estimate, are associated with each group of works. These have been presented in Table 6-1.

The groups of works presented in table 15.1 constitute the time schedule of the planned project. Presented below is the Gantt chart for the project examined. The schedule below presents the planned time of implementation of individual tasks, defining the correlations between them. Apart from the sixteen groups of works, presented in table 15.1, the schedule includes some control points and activities that do not generate costs directly. For the purposes of the method presented, we focused on analysis of deviations, associated with seventeen groups of works specified in table 15.3. Nevertheless, the total time of implementation of the investment is a result of twenty three tasks included in the schedule.
Table 15.1. A tabular breakdown of costs for individual groups of works. Source: own work

<table>
<thead>
<tr>
<th>No.</th>
<th>Name</th>
<th>Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>APARTMENT BUILDING</strong></td>
<td><strong>28 346 000</strong></td>
</tr>
<tr>
<td>1</td>
<td>Preparatory, background works</td>
<td>450 000</td>
</tr>
<tr>
<td>2</td>
<td>Earthworks</td>
<td>1 640 000</td>
</tr>
<tr>
<td>3</td>
<td>Raw state – underground level</td>
<td>2 250 000</td>
</tr>
<tr>
<td>4</td>
<td>Raw state – above ground part</td>
<td>5 300 000</td>
</tr>
<tr>
<td>5</td>
<td>Brickworks – external walls</td>
<td>950 000</td>
</tr>
<tr>
<td>6</td>
<td>Brickworks – internal walls</td>
<td>825 000</td>
</tr>
<tr>
<td>7</td>
<td>Roof insulation</td>
<td>1 320 000</td>
</tr>
<tr>
<td>8</td>
<td>Windows</td>
<td>3 120 000</td>
</tr>
<tr>
<td>9</td>
<td>External plaster + heat insulation</td>
<td>1 950 000</td>
</tr>
<tr>
<td>10</td>
<td>Wall lining – clinker brick</td>
<td>823 000</td>
</tr>
<tr>
<td>11</td>
<td>Interior finishing works</td>
<td>4 220 000</td>
</tr>
<tr>
<td>12</td>
<td>Passenger elevators</td>
<td>758 000</td>
</tr>
<tr>
<td>13</td>
<td>Power supply and low current installations</td>
<td>1 800 000</td>
</tr>
<tr>
<td>14</td>
<td>Sanitary installations</td>
<td>1 350 000</td>
</tr>
<tr>
<td>15</td>
<td>Building service lines</td>
<td>260 000</td>
</tr>
<tr>
<td>16</td>
<td>External works</td>
<td>1 150 000</td>
</tr>
<tr>
<td>17</td>
<td>Preparation of the facility for final acceptance</td>
<td>180 000</td>
</tr>
</tbody>
</table>
Fig. 15.1. Gantt chart for the project planned
Source: own work
During implementation of the project, there are various deviations from the cost or time planned. The objective of this article is to present a method of determining these prior to commencement of the project. It was assumed that information in this regard would be presented by experts in form of three answers to the following questions:

1. What is the most probable cost/time value for task „x” and its probability (level of membership)?
2. What is the minimum cost/time value for task „x” and its probability (level of membership)?
3. What is the maximum cost/time value for task „x” and its probability (level of membership)?

For the needs of analysis, it was assumed that knowledge on deviations was obtained from 3 independent experts, using the issues associated with fuzzy modelling. At the same time, it was assumed that the probability of occurrence of deviation would be expressed as the so-called membership level. In fuzzy modelling (inference), three stages of the operating block can be distinguished: fuzzification, inference – creation of the resulting membership function, defuzzification – sharpening of the fuzzy set. The diagram has been presented in figure 15.2.

---

**Fig. 15.2. A fuzzy model diagram**
Source: own work
The input model data consists of answers to three questions from three experts. The first stage of the operating block is fuzzification. It is based on presentation of input data (information) in form of fuzzy sets. For the needs of this study, it was assumed that the input function of membership of the fuzzy set has the form of a linear piecewise function. The space of sets corresponding with one task of the investment has been presented below:

![Diagram showing membership level (probability) vs. cost with three experts' opinions, Expert 1, Expert 2, Expert 3, cost levels K1min, K1, K1max, K2min, K2, K2max, K3min, K3, K3max, and membership levels p1, p2, p3.]

**Fig. 15.3. Representation of input information (expert opinions) in form of fuzzy sets**
Source: own work

Each of the three experts has presented the most probable cost (K1, K2, K3) and its probability (p1, p2, p3). Apart from this, the experts gave answers concerning the extreme values, that is (K1min, K2min, K3min, K1max, K2max, K3max) and the corresponding probability values (p1min, p2min, p3min, p1max, p2max, p3max). As it has been mentioned, the probability of emergence of a specific cost was expressed by the membership level. The chart also shows
the Kzal, or the assumed cost. Analogously, the fuzzification process was conducted for the time of implementation of each group of works. This representation of expert knowledge presents three fuzzy sets, in which the central point is the most probable value, and the extreme values are, accordingly, the acceptable minimum and maximum values. Figure 15.3 depicts the fuzzy representation of input information for only one task (group of works), e.g. the preparatory/ background works. In table 15.2, input data on cost deviations for all investment tasks can be found.

In the fuzzy sets theory, Fig. 15.3 simply presents 3 fuzzy sets. This is input information. For further use of the knowledge of three experts, information presented by them was consolidated to one set representing the cost (time) deviations for a given task. In other words, the so-called resulting membership function was presented. The process transforming several input functions into a resulting function in fuzzy modelling is known as inference. There are many operators used to determine the resulting membership function. In this article, we used the arithmetic average operator. The value of the resulting membership function is equal to the average level of membership of each input set, which can be recorded according to formula 15.1:

\[
    f_{\text{wynik}}(k) = \frac{\sum_{i=1}^{n} f_i(k)}{n},
\]

where:

- \( f_{\text{wynik}}(k) \) - the resulting membership function (function determining the probability of occurrence of events),
- \( k \) – function argument – cost or time,
- \( n \) number of experts (number of fuzzy sets), \( n=3 \),
- \( i=1, 2, \ldots, n \).

Upon the assumptions made, the formula can be recorded as follows:

\[
    f_{\text{wynik}}(k) = \frac{f_{\text{EKS1}}(k) + f_{\text{EKS2}}(k) + f_{\text{EKS3}}(k)}{3},
\]

where:

- \( f_{\text{EKS1}}(k) \) – membership function determined on the basis of data from expert 1,
- \( f_{\text{EKS2}}(k) \) – membership function determined on the basis of data from expert 2,
- \( f_{\text{EKS3}}(k) \) – membership function determined on the basis of data from expert 3.
Table 15.2. A breakdown of input cost data for all investment tasks
Source: own work

<table>
<thead>
<tr>
<th>No.</th>
<th>Kz1</th>
<th>Kam1</th>
<th>pwm1</th>
<th>Kz2</th>
<th>Kam2</th>
<th>pwm2</th>
<th>Kz3</th>
<th>Kam3</th>
<th>pwm3</th>
<th>Kz4</th>
<th>Kam4</th>
<th>pwm4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>450 000</td>
<td>315 000</td>
<td>0.65</td>
<td>562 500</td>
<td>0.90</td>
<td>382 000</td>
<td>0.85</td>
<td>270 000</td>
<td>0.70</td>
<td>493 000</td>
<td>0.80</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1 160 000</td>
<td>1 350 000</td>
<td>0.65</td>
<td>1 476 000</td>
<td>0.70</td>
<td>1 558 000</td>
<td>0.60</td>
<td>1 394 000</td>
<td>0.60</td>
<td>2 050 000</td>
<td>0.80</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2 250 000</td>
<td>1 462 500</td>
<td>0.60</td>
<td>1 800 000</td>
<td>0.90</td>
<td>1 912 500</td>
<td>0.90</td>
<td>1 462 500</td>
<td>0.70</td>
<td>1 687 500</td>
<td>0.85</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3 300 000</td>
<td>3 445 000</td>
<td>0.60</td>
<td>5 565 000</td>
<td>0.95</td>
<td>6 095 000</td>
<td>0.70</td>
<td>3 975 000</td>
<td>0.70</td>
<td>4 505 000</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5 500 000</td>
<td>1 045 000</td>
<td>0.90</td>
<td>1 092 500</td>
<td>0.95</td>
<td>1 146 000</td>
<td>0.90</td>
<td>1 092 500</td>
<td>0.65</td>
<td>1 187 500</td>
<td>0.85</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>8 250 000</td>
<td>618 750</td>
<td>0.65</td>
<td>825 000</td>
<td>0.70</td>
<td>948 750</td>
<td>0.65</td>
<td>364 750</td>
<td>0.90</td>
<td>918 750</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1 350 000</td>
<td>792 000</td>
<td>0.80</td>
<td>1 056 000</td>
<td>0.90</td>
<td>1 188 000</td>
<td>0.60</td>
<td>1 254 000</td>
<td>0.75</td>
<td>1 584 000</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>3 120 000</td>
<td>2 964 000</td>
<td>0.75</td>
<td>3 432 000</td>
<td>0.75</td>
<td>3 744 000</td>
<td>0.65</td>
<td>2 184 000</td>
<td>0.85</td>
<td>3 120 000</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>1 950 000</td>
<td>1 755 000</td>
<td>0.75</td>
<td>2 145 000</td>
<td>0.80</td>
<td>2 242 500</td>
<td>0.60</td>
<td>1 467 000</td>
<td>0.95</td>
<td>2 145 000</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>8 250 000</td>
<td>658 400</td>
<td>0.65</td>
<td>1 028 750</td>
<td>0.75</td>
<td>1 069 000</td>
<td>0.65</td>
<td>344 950</td>
<td>0.60</td>
<td>699 550</td>
<td>0.85</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>4 220 000</td>
<td>3 798 000</td>
<td>0.60</td>
<td>4 842 000</td>
<td>0.75</td>
<td>5 064 000</td>
<td>0.75</td>
<td>3 587 000</td>
<td>0.70</td>
<td>4 220 000</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>750 000</td>
<td>492 700</td>
<td>0.75</td>
<td>795 900</td>
<td>0.85</td>
<td>871 700</td>
<td>0.70</td>
<td>871 700</td>
<td>0.60</td>
<td>947 500</td>
<td>0.80</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>1 800 000</td>
<td>1 440 000</td>
<td>0.80</td>
<td>1 820 000</td>
<td>0.80</td>
<td>2 070 000</td>
<td>0.60</td>
<td>1 440 000</td>
<td>0.65</td>
<td>1 559 000</td>
<td>0.85</td>
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<td>14</td>
<td>1 350 000</td>
<td>1 080 000</td>
<td>0.70</td>
<td>1 147 500</td>
<td>0.70</td>
<td>1 147 500</td>
<td>0.65</td>
<td>810 000</td>
<td>0.70</td>
<td>1 315 000</td>
<td>0.70</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>260 000</td>
<td>182 000</td>
<td>0.65</td>
<td>208 000</td>
<td>0.80</td>
<td>260 000</td>
<td>0.80</td>
<td>182 000</td>
<td>0.70</td>
<td>299 000</td>
<td>0.85</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>4 150 000</td>
<td>977 500</td>
<td>0.65</td>
<td>1 380 000</td>
<td>0.80</td>
<td>1 495 000</td>
<td>0.70</td>
<td>805 000</td>
<td>0.65</td>
<td>1 035 000</td>
<td>0.80</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>180 000</td>
<td>117 000</td>
<td>0.65</td>
<td>144 000</td>
<td>0.85</td>
<td>171 000</td>
<td>0.75</td>
<td>171 000</td>
<td>0.60</td>
<td>189 000</td>
<td>0.85</td>
<td></td>
</tr>
</tbody>
</table>
Using the MS-Excel spreadsheet, the resulting function was established for each investment task, presenting the potential risk information. Figures 15.4 and 15.5 below present the operation of inference, using as an example the first group of works in the project or „Preparatory, background works”

**Fig. 15.4. Input functions – opinions of three Experts concerning risk for the task „Preparatory, background works”**
Source: own work

**Fig. 15.5. Resulting risk function for task „Preparatory, background works”**
Source: own work
The defuzzification process leads to obtaining of an acute value, which reflects a given fuzzy set. Having the resulting function of the expert opinion, we established two points, dividing the set of results into the optimistic and pessimistic part. These points were established using the centre of mass method. The acute value was calculated on the basis of formula 15.3:

\[
k_{\text{wyn}} = \frac{\int k \cdot u_{\text{wyn}}(k) dk}{\int u_{\text{wyn}}(k) dk},
\]  

(15.3)

where:

- \( k_{\text{wyn}} \) – the acute costs value (input value),
- \( u_{\text{wyn}}(k) \) – the resulting membership function,
- \( k \) – the cost (argument) of the membership function.

In the presented case, the resulting membership function is in form of a broken line. The centre of mass of the broken line, e.g. ABCD, was determined by replacing each line section with a material point, placed in the middle of the section, of the mass equal to the section length. The coordinates of the centre of mass of the broken ABCD were determined on the basis of formulas 15.4 and 15.5. According to Fig. 15.6 symbols \( d_1, d_2, d_3 \) represent lengths of sections AB, BC, CD, and \( S_1(k_1,u_1), S_2(k_2,u_2), S_3(k_3,u_3) \) are centres of these sections.

\[
k_0 = \frac{d_1k_1 + d_2k_2 + d_3k_3}{d_1 + d_2 + d_3},
\]  

(15.4)

\[
u_0 = \frac{d_1u_1 + d_2u_2 + d_3u_3}{d_1 + d_2 + d_3}.
\]  

(15.5)
Fig. 15.6. Graphic representation of determination of the centre of mass of a broken line.
Source: own work

On the basis of the above formulas and, using the Excel spreadsheet for each group of works, optimistic and pessimistic centres of mass were established. For the first group of costs, the “Preparatory, background works”, were presented in Fig. 15.7.
Fig. 15.7. Graphic representation of establishing of the centre of mass for the first group of costs of „Preparatory, background works”
Source: own work

The same operations, that is, fuzzification, inference, defuzzification were conducted for all groups of investment costs. The results obtained were presented in tab. 15.3 for cost values and in tab 15.4 for time values.
Tab. 15.3. A breakdown of pessimistic and optimistic cost values with probability

Source: own work

<table>
<thead>
<tr>
<th>No.</th>
<th>Name</th>
<th>Est cost</th>
<th>Opt cost</th>
<th>popt</th>
<th>Pes cost</th>
<th>ppes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>APARTMENT BUILDING</td>
<td>28 346 000</td>
<td>22 648 750</td>
<td>0,53</td>
<td>31 438 350</td>
<td>0,46</td>
</tr>
<tr>
<td>1</td>
<td>Preparatory, background works</td>
<td>450 000</td>
<td>360 000</td>
<td>0,69</td>
<td>517 500</td>
<td>0,50</td>
</tr>
<tr>
<td>2</td>
<td>Earthworks</td>
<td>1 640 000</td>
<td>1 394 000</td>
<td>0,57</td>
<td>1 886 000</td>
<td>0,41</td>
</tr>
<tr>
<td>3</td>
<td>Raw state – underground level</td>
<td>2 250 000</td>
<td>1 631 250</td>
<td>0,53</td>
<td>2 081 250</td>
<td>0,40</td>
</tr>
<tr>
<td>4</td>
<td>Raw state – above ground part</td>
<td>5 300 000</td>
<td>3 975 000</td>
<td>0,60</td>
<td>5 565 000</td>
<td>0,54</td>
</tr>
<tr>
<td>5</td>
<td>Brickworks – external walls</td>
<td>950 000</td>
<td>950 000</td>
<td>0,30</td>
<td>1 163 750</td>
<td>0,41</td>
</tr>
<tr>
<td>6</td>
<td>Brickworks – internal walls</td>
<td>825 000</td>
<td>701 250</td>
<td>0,52</td>
<td>1 419 000</td>
<td>0,29</td>
</tr>
<tr>
<td>7</td>
<td>Roof insulation</td>
<td>1 320 000</td>
<td>990 000</td>
<td>0,45</td>
<td>1 419 000</td>
<td>0,29</td>
</tr>
<tr>
<td>8</td>
<td>Windows</td>
<td>3 120 000</td>
<td>2 496 000</td>
<td>0,52</td>
<td>3 432 000</td>
<td>0,47</td>
</tr>
<tr>
<td>9</td>
<td>External plaster + heat insulation</td>
<td>1 950 000</td>
<td>1 706 250</td>
<td>0,54</td>
<td>2 242 500</td>
<td>0,75</td>
</tr>
<tr>
<td>10</td>
<td>Wall lining – clinker brick</td>
<td>823 000</td>
<td>658 400</td>
<td>0,52</td>
<td>946 450</td>
<td>0,43</td>
</tr>
<tr>
<td>11</td>
<td>Interior finishing works</td>
<td>4 220 000</td>
<td>3 376 000</td>
<td>0,45</td>
<td>4 642 000</td>
<td>0,44</td>
</tr>
<tr>
<td>12</td>
<td>Passenger elevators</td>
<td>758 000</td>
<td>625 350</td>
<td>0,50</td>
<td>890 650</td>
<td>0,33</td>
</tr>
<tr>
<td>13</td>
<td>Power supply and low current installations</td>
<td>1 800 000</td>
<td>1 485 000</td>
<td>0,77</td>
<td>1 845 000</td>
<td>0,42</td>
</tr>
<tr>
<td>14</td>
<td>Sanitary installations</td>
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<td>1 012 500</td>
<td>0,55</td>
<td>1 586 250</td>
<td>0,48</td>
</tr>
<tr>
<td>15</td>
<td>Building service lines</td>
<td>260 000</td>
<td>195 000</td>
<td>0,67</td>
<td>286 000</td>
<td>0,51</td>
</tr>
<tr>
<td>16</td>
<td>External works</td>
<td>1 150 000</td>
<td>948 750</td>
<td>0,47</td>
<td>1 322 500</td>
<td>0,48</td>
</tr>
<tr>
<td>17</td>
<td>Preparation of the facility for final acceptance</td>
<td>180 000</td>
<td>144 000</td>
<td>0,37</td>
<td>193 500</td>
<td>0,46</td>
</tr>
</tbody>
</table>
Tab. 15.4. A breakdown of pessimistic and optimistic time values with probability
Source: own work

<table>
<thead>
<tr>
<th>No.</th>
<th>Name</th>
<th>Time</th>
<th>Opt t</th>
<th>popt</th>
<th>Pes t</th>
<th>ppes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Preparatory, background works</td>
<td>488</td>
<td>415</td>
<td>0,48</td>
<td>539</td>
<td>0,47</td>
</tr>
<tr>
<td>2</td>
<td>Earthworks</td>
<td>104</td>
<td>96</td>
<td>0,65</td>
<td>130</td>
<td>0,54</td>
</tr>
<tr>
<td>3</td>
<td>Raw state – underground level</td>
<td>74</td>
<td>59</td>
<td>0,75</td>
<td>83</td>
<td>0,53</td>
</tr>
<tr>
<td>4</td>
<td>Raw state – above ground part</td>
<td>118</td>
<td>86</td>
<td>0,47</td>
<td>124</td>
<td>0,23</td>
</tr>
<tr>
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<td>115</td>
<td>0,27</td>
<td>161</td>
<td>0,46</td>
</tr>
<tr>
<td>6</td>
<td>Brickworks – internal walls</td>
<td>130</td>
<td>104</td>
<td>0,48</td>
<td>75</td>
<td>0,51</td>
</tr>
<tr>
<td>7</td>
<td>Roof insulation</td>
<td>77</td>
<td>58</td>
<td>0,44</td>
<td>75</td>
<td>0,51</td>
</tr>
<tr>
<td>8</td>
<td>Windows</td>
<td>118</td>
<td>91</td>
<td>0,52</td>
<td>127</td>
<td>0,48</td>
</tr>
<tr>
<td>9</td>
<td>External plaster + heat insulation</td>
<td>120</td>
<td>81</td>
<td>0,67</td>
<td>117</td>
<td>0,56</td>
</tr>
<tr>
<td>10</td>
<td>Wall lining – clinker brick</td>
<td>101</td>
<td>83</td>
<td>0,58</td>
<td>116</td>
<td>0,43</td>
</tr>
<tr>
<td>11</td>
<td>Interior finishing works</td>
<td>253</td>
<td>215</td>
<td>0,63</td>
<td>291</td>
<td>0,62</td>
</tr>
<tr>
<td>12</td>
<td>Passenger elevators</td>
<td>93</td>
<td>70</td>
<td>0,54</td>
<td>102</td>
<td>0,34</td>
</tr>
<tr>
<td>13</td>
<td>Power supply and low current installations</td>
<td>323</td>
<td>234</td>
<td>0,46</td>
<td>323</td>
<td>0,38</td>
</tr>
<tr>
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<td>302</td>
<td>0,29</td>
<td>421</td>
<td>0,48</td>
</tr>
<tr>
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<td>140</td>
<td>119</td>
<td>0,50</td>
<td>158</td>
<td>0,52</td>
</tr>
<tr>
<td>16</td>
<td>External works</td>
<td>114</td>
<td>97</td>
<td>0,51</td>
<td>120</td>
<td>0,39</td>
</tr>
<tr>
<td>17</td>
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<td>26</td>
<td>23</td>
<td>0,50</td>
<td>33</td>
<td>0,48</td>
</tr>
</tbody>
</table>

In this analysis, the measure of risk applied was the ratio of the value of deviation to its probability. Thanks to the above analysis of each investment task, it is possible to establish the summarised deviations associated with the entire project. Fig. 15.8 and 15.9 present the resulting estimated, pessimistic and optimistic values of cost and time. They were based on aggregation of individual values for investment tasks, using the MS-Project software to create three scenarios of task implementation: optimistic, estimated and pessimistic. The resulting probability of summarised values has been established as a weighted average.
Fig. 15.8. Graphic representation of risk for the investment cost.
Source: own work

Fig. 15.9. Graphic representation of risk for the investment time.
Source: own work
Thanks to this representation of the cost and time of implementation, the general project risk was determined on the basis of formulas 15.6 and 15.7.

\[
R_{pes}^k = \frac{(K_{pes} - K_{zal})}{1 - p_{pes}} = \frac{31438000 - 28346000}{1 - 0.46} = 0.202
\]

\[
R_{opt}^k = \frac{(K_{zal} - K_{opt})}{1 - p_{opt}} = \frac{28346000 - 22648750}{1 - 0.53} = 0.428
\]

\[
(15.6)
\]

\[
(15.7)
\]

**15.4 CONCLUSIONS**

The method presented provides the general contractor or the investor with knowledge on the potential deviations, cost and time associated risks. This method uses expert knowledge associated with individual stages of works, which are much diversified. Expert knowledge has been used separately for each stage, at the same time obtaining the risk associated with the entire project.

On the basis of quantification of risk, we obtained the possibility of responding quickly to unexpected scenarios. Thanks to this analysis, from the beginning of the investment the contractor or the investor is aware of the potential threats associated with a failure to meet the deadline, or overspending the planned budget.

The analysis proposed makes it possible to control the project further during its implementation. It is possible to use the data obtained to control the project e.g. using the earned value method.
15.5 THE PROCEDURE ALGORITHM – A BLOCK DIAGRAM

Presented below are the general rules of the procedure in form of a block diagram (Fig.15.6).

Fig. 15.6. A block diagram of time-cost analysis of the project planned, taking into account the risk, on the basis of expert knowledge, using fuzzy sets.
Source: own work
CHAPTER 16

LITERATURE


